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Mechanical Engineering Sem-III (Rev) Winter 2010(10-WIN-MECH-III-REV-SOM)

Strength of Materials

Q. 1 Answer any four of the following:—

a) A material has Young's Modulus of 2 x 10^5 N/mm2 and Poisson's Ratio of 0.32. Calculate the Modulus of Rigidity and Bulk Modulus of the material.

Ans: $$E = 2 \times 10^5\ N/mm^2$$ [05]

$$\frac{1}{m} = 0.32$$

We have,

$$E = 2C \times (1 + 1/m)$$

$$2 \times 10^5 = 2C \times (1 + 0.32)$$

$$C = 0.75 \times 10^5\ N/mm^2$$

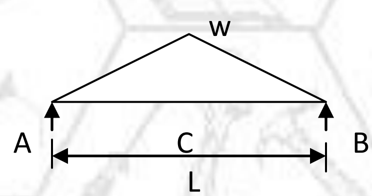
$$E = 3K \times (1 - 2/m)$$

$$2 \times 10^5 = 3K \times (1 - 2 \times 0.32)$$

$$K = 1.85 \times 10^5\ N/mm^2$$

Q. 1 b) Derive the relationship between the rate of loading, shear force and bending moment in a beam.

Ans: [05]



$$\text{Consider simply supported beam AB of span l}\ \ \text{Total load} = \frac{wl}{2}$$

$$Va = Vb = \frac{wl}{4}$$

$$\text{Consider any distance X in AC x from the end A}$$

$$\text{Rate of loading at X} = \frac{x}{\frac{l}{2}} \times w$$

$$\text{External load on AX} = \frac{x}{2} \times \frac{2wx}{l} = \frac{wx^2}{2} \text{ acting at } \frac{x}{3} \text{ from X}$$

$$\text{SF at X} = +\frac{wl}{4} - \frac{wx^2}{l}$$

$$x = 0 \qquad Sx = \frac{wl}{4}$$

$$x = l/2 \qquad Sx = 0$$

$$x = l \qquad Sx = \frac{-wl}{4}$$

$$\text{B.M. at X is given by } Mx = \frac{wlx}{4} - \frac{wx^2}{l} \times \frac{x}{3}$$

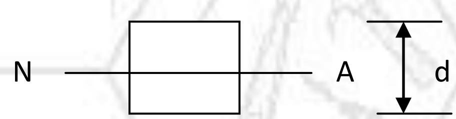
$$xo = 0 \qquad Mx = 0$$

$$xo = \frac{l}{2} \qquad Mx = \frac{wl^2}{12}$$

$$\text{B.M. max} = \text{Total load} \times \frac{\text{span}}{6}$$

Q. 1 c) *For a rectangular beam of depth d, maximum shear stress at the neutral axis was found to be 10 MPa. Find the shear stress at a layer situated at d/4 from the neutral axis.*

Ans: [05]



$$\mathbf{q_{max} = 10\ Mpa}$$

$$\mathbf{q_{max} = \frac{3}{2} \times \frac{S}{bd}}$$

$$\mathbf{10 = \frac{3}{2} \times \frac{S}{bd}}$$

$$\frac{S}{bd} = \frac{20}{3}$$

Now,

$$q = \frac{6S}{bd^3} \times (\frac{d^2}{4} - y^2)$$

$$\text{when } y = \frac{d}{4} \text{ then q is}$$

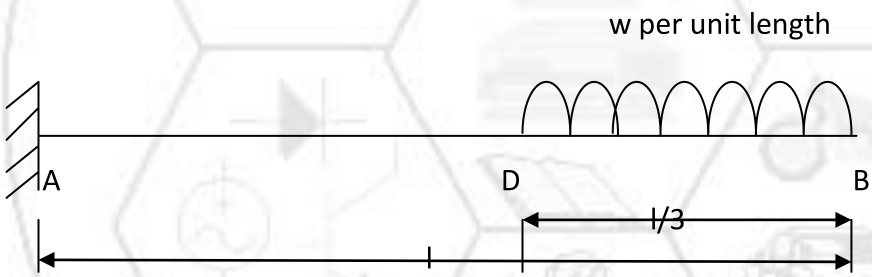
$$q = \frac{9}{8} \times \frac{S}{bd^2}$$

$$q = 7.5 \text{ Mpa}$$

Q. 1 d) A cantilever beam of length L is subjected to a uniformly distributed load of w per unit length for a distance L/3 from the free end. Draw shear force and bending moment diagrams.

Ans: [05]

A cantilever beam of length L is subjected to UDL of w per unit length for a distance L/3 from free end.



Consider any section between D and B distance l/3 from free end B

$$\text{Shear Force at } \frac{l}{3} \qquad SF_{l/3} = w \times l/3$$

$$\text{Shear Force at B} \quad SF_B = 0$$

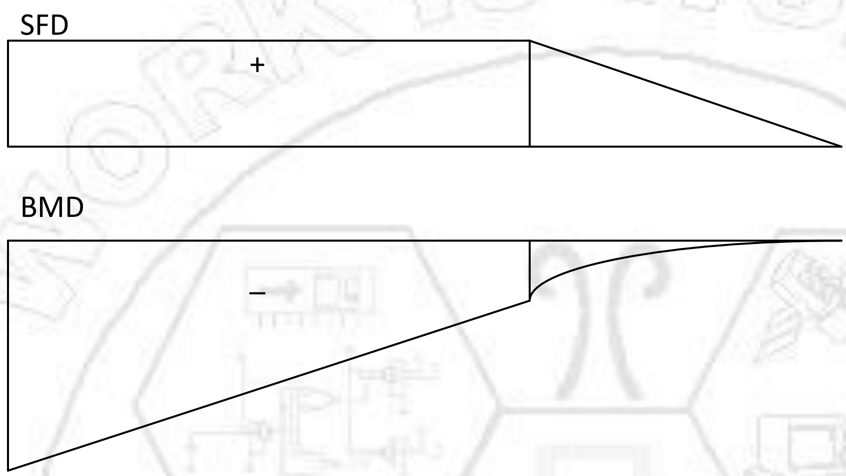
$$\text{Shear Force at D} \quad SF_D = \frac{wl}{3}$$

$$\text{Shear Force at A} \quad SF_A = \frac{wl}{3}$$

$$\text{Bending Moment at B}\quad BM_B = 0$$

$$\text{Bending Moment at D}\quad BM_D = \frac{wl^2}{18}$$

$$\text{Bending Moment at A}\quad BM_A = \frac{5wl^2}{18}$$



Q. 1 e) A short column of external diameter 400 mm and internal diameter 200 mm carries an eccentric load of 80 kN. Find the greatest eccentricity, wNch the load can have without producing tension on the cross section.

Ans: [05]

$$D = 400\ \text{mm}$$

$$d = 200\ \text{mm}$$

$$P = 80\ \text{KN}$$

$$\text{Direct Stress} = \sigma_o = \frac{P}{A} = \frac{80 \times 10^3}{\frac{\pi}{4} \times (400^2 - 200^2)}$$

$$\sigma_o = 0.848\ \text{N/mm}^2$$

$$\text{Stress due to moment } \sigma_b = \frac{M}{Z} = \frac{Pe}{\frac{\pi}{32} \times \frac{(D^4 - d^4)}{D}}$$

$$\text{If tension is just avoided } \sigma_o = \sigma_b$$

$$0.848 = \frac{400 \times 10^3 \times e}{\frac{\pi}{32} \times \frac{(D^4 - d^4)}{D}}$$

$$e = 62.43 \text{ mm.}$$

Q. 1 f) A steel bar of 50 mm x 50 mm in section and 3 m in length is subjected to an axial pull of 140 kN. Calculate the strain energy stored in the bar. Also, find the extension of the bar. Take E=200GPa

Ans: [05]

$$A = 50\text{mm} \times 50\text{mm}$$
$$l = 3\text{m} = 3 \times 1000 \text{ mm}$$
$$P = 140 \text{ KN} = 140 \times 1000 \text{ N}$$
$$E = 200 \text{ GPa} = 200 \times 1000 \text{ MPa}$$
$$\text{Strain energy stored by member} = \sigma^2 \times \frac{AL}{2E}$$
$$U = (P/A)^2 \times \frac{AL}{2E}$$
$$U = (140 \times 1000/(50 \times 50))^2 \times \frac{50 \times 50 \times 3000}{2 \times 200 \times 1000}$$

$$U = 58800 \text{ Nmm}$$
$$\delta l = \frac{Pl}{AE}$$

$$\delta l = \frac{140 \times 1000 \times 3000}{50 \times 50 \times 200 \times 1000}$$

$$\delta l = 0.84 \text{ mm}$$

Q. 2 a)A tube of aluminium 40 mm external diameter and 20mm internal diameter is fitted on a solid steel rod of 20 mm diameter. The composite bar is loaded in compression by an axial load P. Find the stress in aluminium1 when the load is such that the stresses in steel is 70 NfmmZ. Also, find the value of P. Take Es= 2 x 10^5 N/mm^2 Ea= 0. 7 x 10^5 N/mm^2

Ans: [10]

$$D_o = 40 \text{ mm}$$
$$D_i = 20 \text{ mm}$$
$$d = 20 \text{ mm}$$
$$\sigma_{St} = 70 \text{ N/mm}^2$$
$$\text{Area of Aluminium tube} = \frac{\pi}{4} \times ({D_o}^2 - {D_i}^2)$$
$$A_{Al} = \frac{\pi}{4} \times (40^2 - 20^2)$$
$$A_{Al} = 942.47 \text{ mm}^2$$
$$\text{Area of Steel rod} = \frac{\pi}{4} \times d^2$$
$$A_{St} = \frac{\pi}{4} \times 20^2$$
$$A_{St} = 314.15 \text{ mm}^2$$
$$\text{Stress in steel and aluminium be the } \sigma_{St} \text{ and } \sigma_{Al} \text{ resp.}$$
$$\text{strain in steel} = \text{strain in aluminium}$$
$$\frac{\sigma_{St}}{E_{St}} = \frac{\sigma_{Al}}{E_{Al}}$$
$$\sigma_{Al} = \frac{7 \times 10^4}{2 \times 10^5} \times 70$$
$$\sigma_{Al} = 24.5 \text{ N/mm}^2$$
$$\text{Load on steel} + \text{Load on Aluminium} = \text{Total load}$$
$$\sigma_{St} \times A_{St} + \sigma_{Al} \times A_{Al} = P$$
$$70 \times 314.15 + 24.5 \times 942.47 = P$$
$$P = 45076 \text{ N}$$
$$P = 45.076 \text{ KN}$$

Q. 2 b) An unknown weight falls through 8 mm on a collar rigidly attached to the lower end of a vertical bar, 4 m long and 40 mm x 20 mm in section. If the maximum instantaneous extension is known to be 3 mm, what is the corresponcflng stress and the value of the unknown weight?

Take E= 2 x 1o5 NI mm2

Ans: [10]

$$h = 8\text{ mm}$$
$$l = 4000\text{mm}$$
$$A = 40 \times 20\text{ mm}^2$$
$$\delta l = 3\text{ mm}$$
$$E = 2 \times 10^5\text{ N/mm}^2$$
$$\text{Max stress} = \sigma = E \times \text{Max strain} = 2 \times 10^5 \times \frac{3}{4000} = 150\text{ N/mm}^2$$

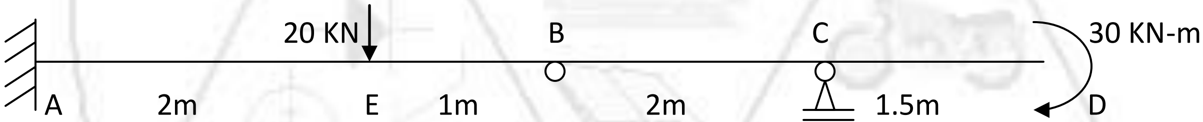
Now load,

Equating loss of potential energy to strain energy stored by the rod, we have

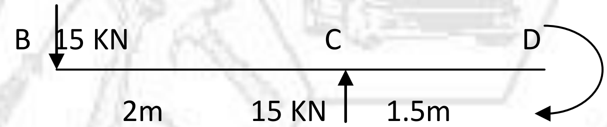
$$P(h + l) = \frac{\sigma^2}{2E} \times Al$$
$$P(8 + 3) = \frac{150^2}{2 \times 10^5} \times 40 \times 20 \times 4000$$
$$P = 16363.63\text{N}$$
$$P = 16.363\text{KN}$$

Q. 3 a) A beam ABCD with an internal hinge at B is loaded as shown in figure. Determine reactions and draw bending moment and shear force diagram.

Ans: [10]



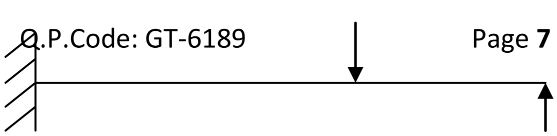
Consider part BD



Taking reaction about B

$$0 = V_c \times 2 - 30$$
$$V_c = 15\text{ KN}$$

Consider part AB



B

A 2m E 1m 15KN

$$V_A + V_B = 20$$

$$V_A = 5 \text{ KN}$$

$$\text{S. F. at A} = 5 \text{ KN}$$

$$\text{S. F. at left of E} = 5 \text{ KN}$$

$$\text{S. F. at right of E} = 5 - 20 = -15 \text{ KN}$$

$$\text{S. F. at left of C} = -15 \text{ KN}$$

$$\text{S. F. at right of C} = -15 + 15 = 0 \text{ KN}$$

$$\text{S. F. at B} = 0 \text{ KN}$$

$$\text{B. M. at A} = 20 \times 2 - 15 \times 3 = -5\text{KNm}$$

$$\text{B. M. at E} = 5 \times 2 = 10 \text{ KNm}$$

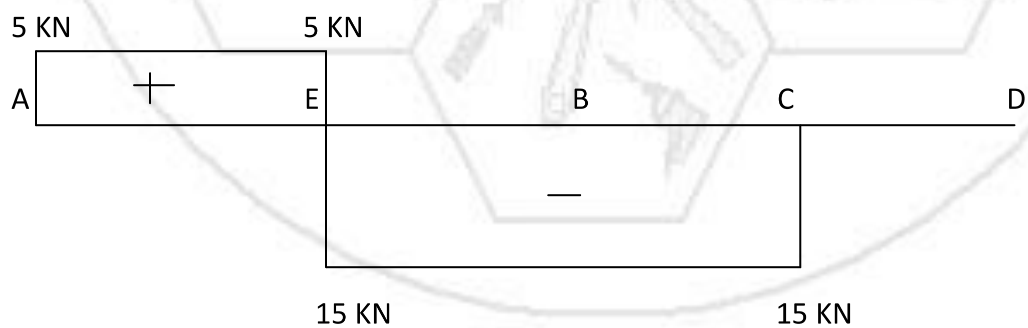
$$\text{B. M. at left side of B} = -20 \times 2 + 15 \times 3 = 5 \text{ KNm}$$

$$\text{B. M. at right side of B} = 0 \text{ KNm}$$

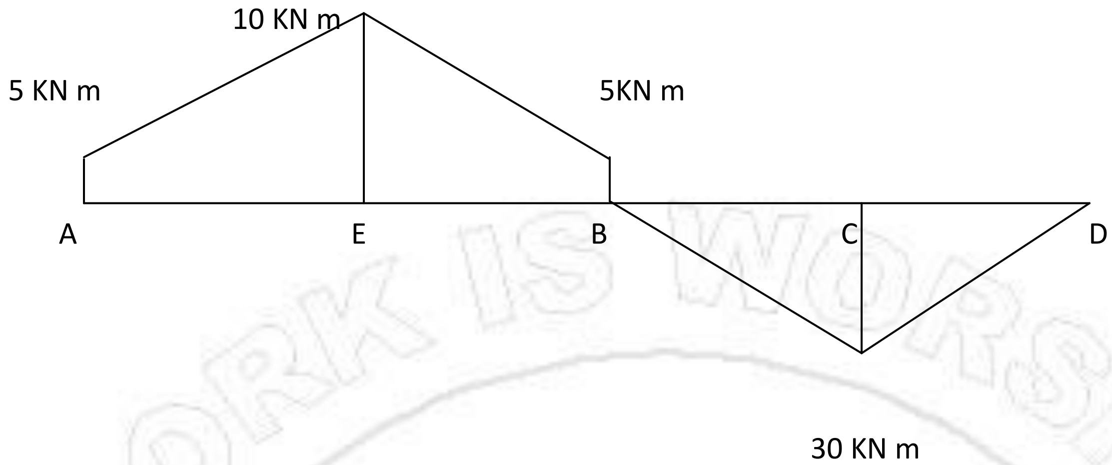
$$\text{B. M. at C} = -30 \text{ KNm}$$

$$\text{B. M. at D} = -30 + 30 = 0 \text{ KNm}$$

S. F. D.



B. M. D.



Q. 3 b) A composite beam having the cross sectional dimensions shown in figure is subjected to a bending moment of 400 kNm. Materials are fastened so that the beam acts as a single unit. Determine the maximum bending stresses in each material. Dimensions are in mm.

Take Es= 2 x 10^5 N/mm^2; Eal= 0.67 x 10^5 N/mm^2; Eci= 1 x 10^5 N/mm^2

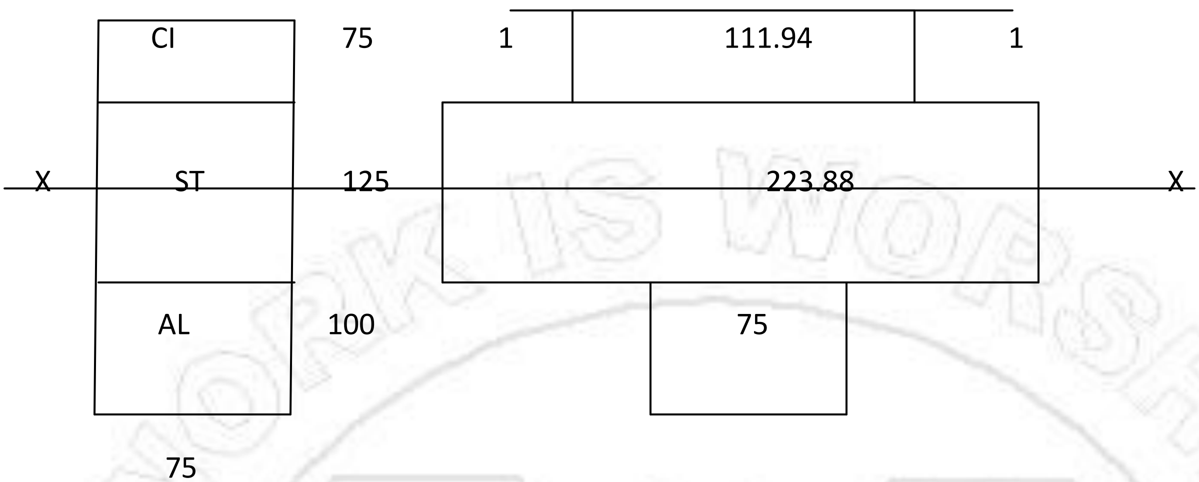
Ans: [10]

$$\text{B.M.} = 400\ \text{Nm}$$

$$E_{st} = 2 \times 10^5 \text{N/mm}^2$$

$$E_{al} = 0.67 \times 10^5 \text{N/mm}^2$$

$$E_{ci} = 1 \times 10^5 \text{N/mm}^2$$



The 75 mm wide CI replaced by an equivalent $= \frac{E_{ci}}{E_{al}} \times 75$

$$= \frac{1 \times 10^5}{0.67 \times 10^5} \times 75$$

$$= 111.94 \text{ mm wide AL strip.}$$

The 75 mm wide ST replaced by an equivalent $= \frac{E_{st}}{E_{al}} \times 75$

$$= \frac{2 \times 10^5}{0.67 \times 10^5} \times 75$$

$$= 223.88 \text{ mm wide AL strip}$$

Component	Area a mm^2	C.G. dist^n from I-I y mm	Ay mm^3	Ay^2 mm^4	I self mm^4
Top rectangle	8395.5	37.5	8395.5*37.5	8395.5*37.5^2	111.94*75^3/12
Middle Rectangle	27985	137.5	27985*137.5	27985*137.5^2	223.88*75^3/12
Bottom rectangle	7500	250	7500*250	7500*250^2	75*75^3/12

	43880.5		6037768.75	1009647578	14442890.63

$$\bar{y} = \frac{\sum ay}{A} = \frac{6037768.75}{43880.5} = 137.5 \text{ mm}$$

$$\text{M.I. of section about axis } 1-1 = \sum \text{Iself} + \sum ay^2$$

$$= 14442890.63 + 1009647578$$

$$= 1024090469 \text{ mm}^4$$

$$I_{x-x} = I_{1-1} - A\bar{y}^2$$

$$I_{x-x} = 1024090469 - 43880.5 \times 137.5^2$$

$$I_{x-x} = 194474765.5 \text{ mm}^4$$

$$\text{Max bending stress in Al} = \frac{400 \times 10^6}{194474765.5} \times 162.5 \times \frac{75}{75}$$

$$\text{Max bending stress in Al} = 334.23 \text{ N/mm}^2$$

$$\text{Max tensile stress in Steel} = \frac{400 \times 10^6}{194474765.5} \times 62.5 \times \frac{223.88}{75}$$

$$\text{Max tensile stress in Steel} = 383.73 \text{ N/mm}^2$$

$$\text{Max compressive stress in Steel} = \frac{400 \times 10^6}{194474765.5} \times 62.5 \times \frac{223.88}{75}$$

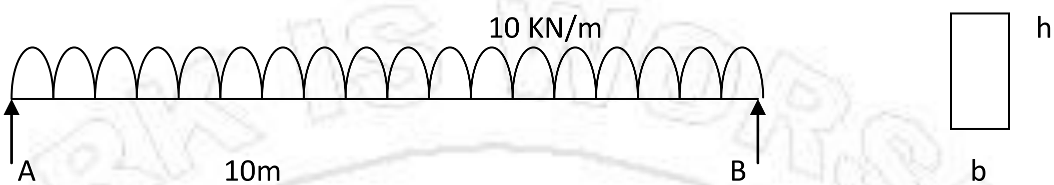
$$\text{Max compressive stress in Steel} = 383.73 \text{ N/mm}^2$$

$$\text{Max compressive stress in C.I.} = \frac{400 \times 10^6}{194474765.5} \times 137.5 \times \frac{111.94}{75}$$

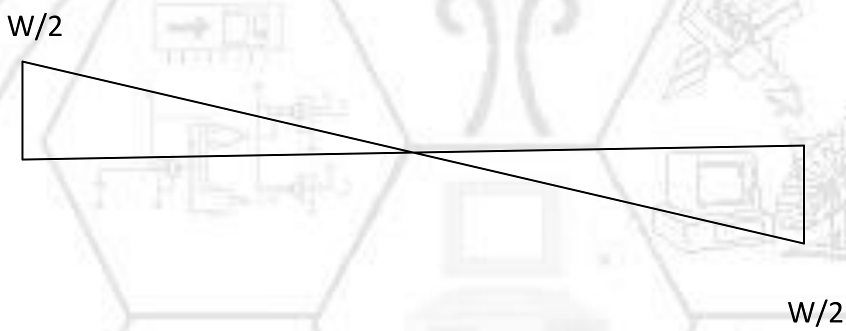
$$\text{Max compressive stress in C.I.} = 422.10 \text{ N/mm}^2$$

Q. 4 a) A timber beam having rectangular cross section is loaded with a uniformly distributed load of 10 kN/m. It is simply supported at its ends for a span of 10 m. If the allowable design *stresses are 10 N/mm2 in bending and 1 N/mm2 in shear, what is the span to depth ratio, such that allowable flexural and shear stresses will occur simultaneously?*

Ans: $W = 10 \times 10 \times 10^3 N$ [10]



SFD



BMD

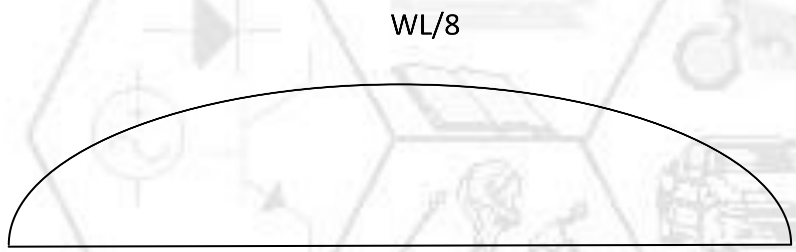


Figure shows B.M. & S.F.

$$S.F._{.max} = \frac{W}{2}$$

$$B.M._{.max} = \frac{WL}{8} \text{ at mid}$$

Design for shear:

$$\tau = \frac{2}{3} \times \frac{\text{S.F.max}}{\text{A}}$$

$$1 = \frac{2}{3} \times \frac{\text{W}/2}{\text{bh}}$$

$$\text{W} = \frac{4}{3} \times \text{bh}$$

Design for bending:

$$\sigma = \frac{\text{Mmax}}{\text{Z}}$$

$$\sigma = \frac{\text{WL}/8}{\text{bh}^2/8}$$

$$10 = \frac{3}{4} \times \frac{\text{WL}}{\text{bh}^2}$$

$$\text{W} = \frac{40}{3} \times \frac{\text{bh}^2}{\text{L}}$$

From above

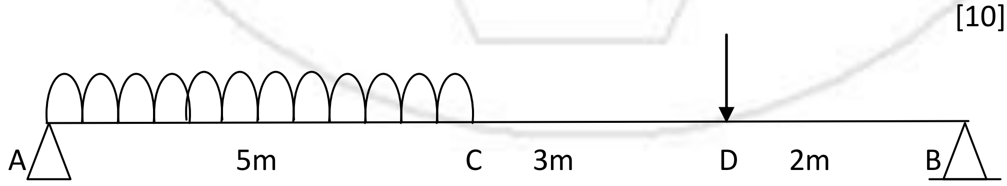
$$\frac{4}{3} \times \text{bh} = \frac{40}{3} \times \frac{\text{bh}^2}{\text{L}}$$

$$1 = \frac{10\text{h}}{\text{L}}$$

$$\frac{\text{L}}{\text{h}} = 10$$

Q. 4 b) For a beam loaded as shown in figure, find slope at points A & B, deflections at points C & D. Also, find the maximum deflection.

Ans: [10]



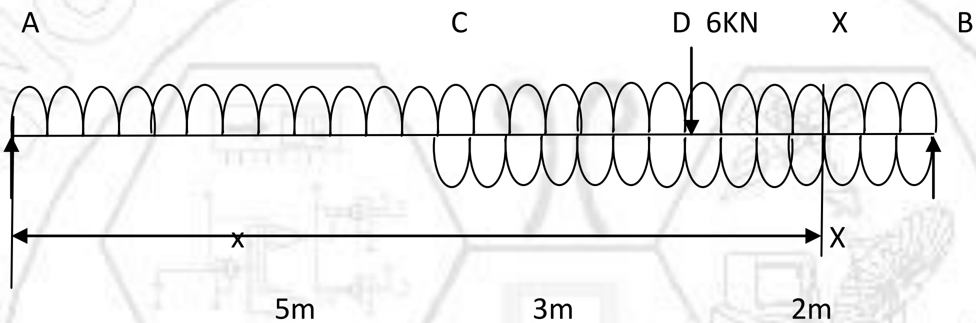
Taking moment about A,

$$0 = 1 \times 5 \times \frac{5}{2} + 6 \times 8 - Rb \times 10$$

$$Rb = \frac{60.5}{10} = 6.05KN$$

$$Ra = 1 \times 5 + 6 - 6.05 = 4.95KN$$

On part CB of given beam assume a UDL 1KN/m applied both from above and below so that both these added loads neutralize each other and net effect remains unchanged



Beam with modified loading

Consider section X-X at a distance x from the support A

$$Mx = EI\frac{d^2y}{dx^2} = 4.95x - 1 \times \frac{x^2}{2} - 6 \times (x-8) + 1 \times \frac{(x-5)^2}{2}$$

$$EI\frac{dy}{dx} = \frac{4.95x^2}{2} - \frac{x^3}{6} - \frac{6 \times (x-8)^2}{2} + \frac{(x-5)^3}{6} + C1$$

$$EIy = \frac{4.95x^3}{6} - \frac{x^4}{24} - \frac{6 \times (x-8)^3}{6} + \frac{(x-5)^4}{24} + C1x + C2$$

$$\text{When } x = 0, y = 0 \therefore C2 = 0$$

$$\text{When } x = 10, y = 0$$

$$0 = 4.95 \times \frac{10^3}{6} - \frac{10^4}{24} - \frac{6(10-8)^3}{6} + \frac{(10-5)^4}{24} + 10C1$$

$$C1 = -42.63$$

$$Hence\ slope\ and\ deflection\ equations\ are$$

$$EI\frac{dy}{dx} = \frac{4.95x^2}{2} - \frac{x^3}{6} - \frac{6(x-8)^2}{2} + \frac{(x-5)^3}{6} - 42.63$$

$$EIy = \frac{4.95x^3}{6} - \frac{x^4}{24} - \frac{6(x-8)^3}{6} + \frac{(x-5)^4}{24} - 42.63x$$

Deflection at C, yc   put x=5m (neglecting square bracket in which the terms are –ve)

$$EIy_c = \frac{(4.95 \times 5000^3)}{6} - \frac{5000^4}{24} - 42.95 \times 5000$$

$$y_c = -2.59 \times \frac{10^{13}}{EI}\ mm\ (downward)$$

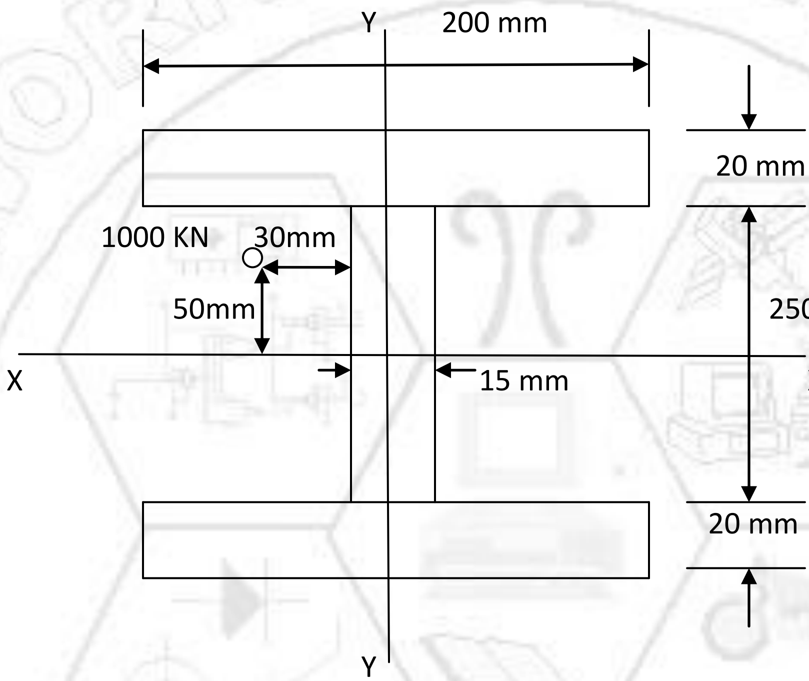
Deflection at D, yd   put x=8m (neglecting square bracket in which the terms are –ve)

$$EIy_d = \frac{(4.95 \times 8000^3)}{6} - \frac{8000^4}{24} - 42.95 \times 8000$$

$$y_d = -1.70 \times \frac{10^{14}}{EI}\ mm\ (downward)$$

Q. 5 a) A rolled steel I-section with top and bottom flanges 200 mm x 20 mm and web 15 mm *wide and 250 mm deep is used as a short column to carry a load of 1000 IcN. If the load line is eccentric SO mm above x-x axis and 30 mm left of y-y axis, find the maximum and minimum stress intensity induced in the section.*

Ans: [10]



$$Area\ of\ section = 200 \times 20 \times 2 + 250 \times 15 = 11750\ mm^2$$

$$Section\ modulus\ about\ xx\ axis = \frac{I_{xx}}{Y_{max}xx} = \frac{\left[\frac{200 \times 290^3}{12} - \frac{185 \times 250^3}{12}\right]}{145}$$

$$Z_{xx} = 1137931.034\ mm^3$$

$$Section\ modulus\ about\ yy\ axis = \frac{I_{yy}}{Y_{max}yy} = \frac{\left[\frac{290 \times 200^3}{12} - \frac{250 \times 185^3}{12}\right]}{100}$$

$$Z_{yy} = 618300\ mm^3$$

$$\sigma_{xx} = \frac{M_{xx}}{Z_{xx}} = \frac{1000 \times 1000 \times 50}{1137931.034} = 43.93\frac{N}{mm^2}$$

$$\sigma_{yy} = \frac{M_{yy}}{Z_{yy}} = \frac{1000 \times 1000 \times 50}{618300} = 80.86\frac{N}{mm^2}$$

$$Direct\ stress = \sigma_o = \frac{P}{A} = 1000 \times \frac{1000}{11750} = 85.10\ \frac{N}{mm^2}$$

$$Bending\ stress = \sigma_b = \sqrt{{\sigma_{xx}}^2 + {\sigma_{yy}}^2}$$

$$\sigma_b = \sqrt{43.93^2 + 85.10^2}$$

$$\sigma_b = 95.76\ \frac{N}{\mathrm{mm}^2}$$

$$\sigma_{max} = \sigma_b + \sigma_o = 95.76 + 85.10$$

$$\sigma_{max} = 180.86\ \frac{N}{mm^2}$$

$$\sigma_{min} = \sigma_b - \sigma_o = 95.76 - 85.10$$

$$\sigma_{max} = 10.66\ \frac{N}{mm^2}$$

Q. 5 b) From the following data, determine the thickness of cast iron column. Assume both the ends of the column are fixed.

Length of the column= 3 m

External diameter= 200 mm

Safe working load= 600 kN

Factor of Safety= 5

Ultimate compressive stress 570 N/mm2

Rankine constant= 1/ 1600

Ans: [10]

$$l = 3m = 3000\ mm$$

$$D = 200\ mm$$

$$P = 600\ KN$$

$$FOS = 5$$

$$\sigma_c = 570\ \frac{N}{mm^2} = 570 \times 10^6\ \frac{N}{m^2}$$

$$a = \frac{1}{1600}$$

$$le = \frac{l}{2} = \frac{3}{2} = 1.5$$

$$\text{Cripping load} = P \times FOS = 600 \times 5 = 3000\ KN$$

$$\text{Useing Rankine's formula, we get}$$

$$P_{Rankine} = \frac{\sigma_c \times A}{1 + a \times \left(\frac{le}{K}\right)^2}$$

$$K = \frac{I}{A} = \frac{\pi \times \frac{(D^4 - d^4)}{64}}{\pi \times \frac{(D^2 - d^2)}{4}} = \frac{(D^2 + d^2)}{16}$$

$$3000 \times 1000 = \frac{570 \times 10^6 \times \pi \times \frac{(0.2^2 - d^2)}{4}}{1 + \frac{1}{1600} \times \left(\frac{1.5}{\frac{(D^2 + d^2)}{16}}\right)^2}$$

After solving we get

$$d^4 + 0.0068 \times d^2 - 0.001175 = 0$$

$$d^2 = 0.03104$$

$$d = 0.1761 \text{ m}$$

$$d = 176.1 \text{ mm}$$

$$t = \frac{D - d}{2} = \frac{200 - 176.1}{2}$$

$$t = 11.95\text{mm.}$$

Q. 6 a) A hollow shaft of 4 m long is to transmit 150 kW power at 150 RPM. If the total angle of twist in this length and the allowable shear stress are not to exceed 2.5° and 60 N/mm2 respectively, determine the inside and outside diameters. Take G= 0.8 x 10^5 N/mm^2

Ans: [10]

$$l = 4 \text{ m} = 4000 \text{ mm}$$

$$P = 150\text{Kw}$$

$$N = 150 \text{ rpm}$$

$$\theta = 2.5^0 = 2.5 \times \frac{\pi}{180} = 0.04363 \text{ rad}$$

$$\tau = 60 \ \frac{N}{mm^2}$$

$$G = 0.8 \times 10^5 \frac{N}{mm^2}$$

$$P = \frac{2\pi NT}{60}$$

$$T = \frac{150 \times 60}{2\pi 150} = 9.54 \text{ KN} - \text{m} = 9.54 \times 10^6 \text{ N} - \text{mm.}$$

Torque transmitted,

$$T = \frac{\pi}{16} \times \tau \times \frac{D^4 - d^4}{D}$$

$$9.54 \times 10^6 = \frac{\pi}{16} \times 60 \times \frac{D^4 - d^4}{D}$$

$$D^4 - d^4 = 809780.35D \quad \longrightarrow \quad A$$

$$\frac{T}{I_P} = \frac{C\theta}{l}$$

$$\frac{9.54 \times 10^6}{\frac{\pi}{32} \times \frac{D^4 - d^4}{1}} = \frac{0.8 \times 10^5 \times 0.04363}{4000}$$

$$\frac{9.54 \times 10^6 \times 4000 \times 32}{\pi \times 0.8 \times 10^5 \times 0.04363} = D^4 - d^4$$

$$D^4 - d^4 = 111361038.3 \quad \longrightarrow \quad B$$

from A and B

$$809780.35D = 111361038.3$$

$$D = 137.52 \text{ mm}$$

putting in B

$$d = 125.27 \text{ mm}$$

Q. 6 b) A cylindrical shell is 3 m long, 1 m internal diameter and 15 mm thickness. If it is subjected to an internal pressure of 1.5 N/mm2, calculate the maximum intensity of shear stress induced and the change in volume of the shell. Take, E 2.04 x 10^5 N/mm^2

Poison's ratio= 0.3

Ans: *[10]*

$$l = 3m = 3000mm$$

$$d = 1m = 1000mm$$

$$t = 15mm$$

$$\sigma = 1.5\ N/mm^2$$

$$E = 2.04 \times 10^5 \frac{N}{mm^2}$$

$$\frac{1}{m} = 0.3$$

$$\text{Hoop stress} = \sigma_1 = \frac{\sigma d}{2t} = \frac{1.5 \times 1000}{2 \times 15} = 50\ N/mm^2$$

$$\text{Longitudinal stress} = \sigma_2 = \frac{\sigma d}{4t} = \frac{1.5 \times 1000}{4 \times 15} = 25\ N/mm^2$$

$$\text{Circumferntial strain} = e_1 = \frac{1}{E} \times (\sigma_1 - \frac{\sigma_2}{m})$$

$$e_1 = \frac{1}{2.04 \times 10^5} \times (50 - 25 \times 0.3)$$

$$e_1 = 2.0833 \times 10^{-4}$$

$$\text{Longitudinal strain} = e_2 = \frac{1}{E} \times (\sigma_2 - \frac{\sigma_1}{m})$$

$$e_2 = \frac{1}{2.04 \times 10^5} \times (25 - 50 \times 0.3)$$

$$e_2 = 4.90 \times 10^{-4}$$

$$\text{Volumetric strain} = e_v = 2e_1 + e_2$$

$$e_v = 2 \times 2.0833 \times 10^{-4} + 4.90 \times 10^{-4}$$

$$e_v = 9.0666 \times 10^{-4}$$

$$\text{Change in volume} = e_v V$$

$$d_v = 9.0666 \times 10^{-4} \times \frac{\pi}{4} \times (1030^2 - 1000^2) \times 3000$$

$$d_v = 71746.12 \text{ mm}^3$$

$$\text{Principal stresses} = \frac{\sigma_1 + \sigma_2}{2} \pm \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + q^2}$$

$$P_1, P_2 = \frac{50 + 25}{2} \pm \sqrt{\left(\frac{50 - 25}{2}\right)^2 + q^2}$$

$$P_1 = 37.5 \pm \sqrt{156.25 + q^2}$$

$$P_2 = 37.5 \pm \sqrt{156.25 + q^2}$$

$$\text{Max shear stress} = \frac{P_1 - P_2}{2}$$

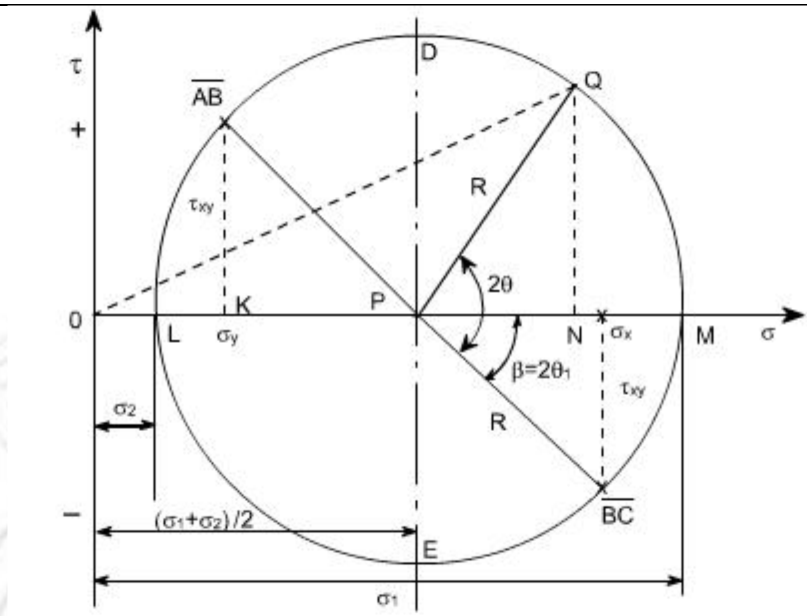
$$\text{Max shear stress} = \sqrt{156.25 + q^2} \text{N/mm}^2$$

Q. 7 a) At a point in an elastic material under strain, there are normal stresses of SON/mm2 and 30 N/mm1 at right angles to each other and a shear stress of 25 N/mm2. Find graphically or otherwise, the principal stresses and the position of principal plane, if

(1) Both 50 N/mm2 and 30 N/mm2 are tensile

(ii) 50 N/mm2 is tensile and 30 N/mm2 is compressive.

Ans: Graphical method: [10]



Analytical method:

1) Both stresses are tensile types

$$\sigma = 50\,\frac{N}{mm^2}$$

$$\sigma' = 30\,\frac{N}{mm^2}$$

$$\tau = 25\,\frac{N}{mm^2}$$

Principal stresses are given by

$$\sigma_1, \sigma_2 = \frac{\sigma + \sigma'}{2} \pm \sqrt{\frac{(\sigma - \sigma')^2}{2} + \tau^2}$$

$$\sigma_1, \sigma_2 = \frac{50 + 30}{2} \pm \sqrt{\frac{(50 - 30)^2}{2} + 25^2}$$

$$\sigma_1 = 66.92\ \frac{N}{mm^2}$$

$$\sigma_2 = 13.09\ \frac{N}{mm^2}$$

Positions of principal planes

$$\tan 2\theta = \frac{2\tau}{(\sigma - \sigma')} = \frac{2 \times 25}{50 - 30} = 2.5$$

$$2\theta = \tan^{-1} 2.5$$

$$2\theta = 68.19 \text{ and } 248.19$$

$$\theta = 34.09 \text{ and } 124.09$$

$$\theta_1 = 34.09^0 \text{ and } \theta_2 = 124.09^0$$

2) σ is tensile stresses and σ′ is compressive types

$$\sigma = 50 \frac{N}{mm^2}$$

$$\sigma' = -30 \frac{N}{mm^2}$$

$$\tau = 25 \frac{N}{mm^2}$$

Principal stresses are given by

$$\sigma_1, \sigma_2 = \frac{\sigma + \sigma'}{2} \pm \sqrt{\frac{(\sigma - \sigma')^2}{2} + \tau^2}$$

$$\sigma_1, \sigma_2 = \frac{50 - 30}{2} \pm \sqrt{\frac{(50 + 30)^2}{2} + 25^2}$$

$$\sigma_1 = 57.16 \ \frac{N}{mm^2}$$

$$\sigma_2 = -37.16 \ \frac{N}{mm^2}$$

Positions of principal planes

$$\tan 2\theta = \frac{2\tau}{(\sigma - \sigma')} = \frac{2 \times 25}{50 + 30} = 0.625$$

$$2\theta = \tan^{-1} 2.5$$

$$2\theta = 32 \text{ and } 212$$

$$\theta = 16 \text{ and } 106$$

$$\theta_1 = 16^0 \text{ and } \theta_2 = 106^0$$

Q. 7 b) List the assumptions made in theory of pure bending and hence derive the formula,

[06]

Ans:

Simple Bending Theory

Assumptions:

1. Beam is initially straight, and has a constant cross-section.

2. Beam is made of homogeneous material and the beam has a longitudinal plane of symmetry.

3. Resultant of the applied loads lies in the plane of symmetry.

4. The geometry of the overall member is such that bending not buckling is the primary cause of failure.

5. Elastic limit is nowhere exceeded and E' is same in tension and compression.

6. Plane cross - sections remains plane before and after bending.

Consider now fiber AB in the material, at a distance y from the N.A, when the beam bends this will stretch to A'B'

Therefore,

$$\text{strain in fibre AB} = \frac{\text{change in length}}{\text{orginal length}}$$

$$= \frac{A'B' - AB}{AB} \qquad \text{But } AB = CD \text{ and } CD = C'D'$$

refer to fig1(a) and fig1(b)

$$\therefore \text{strain} = \frac{A'B' - C'D'}{C'D'}$$

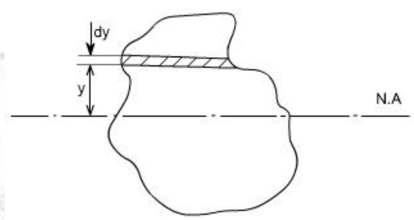
Since CD and C'D' are on the neutral axis and it is assumed that the Stress on the neutral axis zero. Therefore, there won't be any strain on the neutral axis

$$= \frac{(R+y)\theta - R\theta}{R\theta} = \frac{R\theta + y\theta - R\theta}{R\theta} = \frac{y}{R}$$

However $\frac{\text{stress}}{\text{strain}} = E$ where E = Young's Modulus of elasticity

Therefore, equating the two strains as obtained from the two relations i.e,

$$\frac{\sigma}{E} = \frac{y}{R} \text{ or } \frac{\sigma}{y} = \frac{E}{R} \qquad \text{.............(1)}$$



Consider any arbitrary a cross-section of beam, as shown above now the strain on a fibre at a distance �y' from the N.A, is given by the expression

$$\sigma = \frac{E}{R} y$$

if the shaded strip is of area 'dA'
then the force on the strip is

$$F = \sigma\,\delta A = \frac{E}{R} y\,\delta A$$

Moment about the neutral axis would be $= F.y = \frac{E}{R} y^2 \delta A$

The toatl moment for the whole cross-section is therefore equal to

$$M = \sum \frac{E}{R} y^2\,\delta A = \frac{E}{R} \sum y^2 \delta A$$

Now the term $\sum y^2 \delta A$ is the property of the material and is called as a second moment of area of the cross-section and is denoted by a symbol I.

Therefore

$$M = \frac{E}{R} I \qquad \ldots\ldots\ldots(2)$$

combining equation 1 and 2 we get

$$\boxed{\frac{\sigma}{y} = \frac{M}{T} = \frac{E}{R}}$$

This equation is known as the Bending Theory Equation.

Q. 7 b)ii) Define and explain the following with illustrations.

Ans:
I] Principal planes: It has been observed that at any point in a strained material, there are three planes, mutually perpendicular to each other, which carry direct stress only and no shear stress. These perpendicular planes which have no shear stress are called Principal planes. Principal planes that contains maximum and minimum stresses.
Principal stress: The magnitude of direct stress, across a principal plane is called Principal stress. The maximum and minimum stresses, also called as principal stresses. [02]

II] Principal axes: In the most general case of stress, we can determine three coordinate axes a, b and c called the Principal axes. [02]