Fluid Mechanics

Q. 1. 1) *A 150 mm diameter pipe reduces in diameter abruptly to 100 mm. if the pipe carries water at 30 liters/sec. calculates the pressure loss across the contraction and expresses this as a percentage of the pressure loss to be expected if the flow were reversed. Take the coefficient of contraction as 0.6*

Given data,

Diameter of large pipe D_1= *150 mm= 0.15 m*

Diameter of large pipe D_2= *100 mm= 0.10 m*

Discharge Q= 30 lit/sec= 0.03 m^3/s

Coefficient of contraction = 0.6

$$Area\ of\ large\ pipe = A_1 = \frac{\pi}{4}{D_1}^2 = \frac{\pi}{4} \times 0.15^2 = 0.01767\ m^2$$

$$Area\ of\ Small\ pipe = A_2 = \frac{\pi}{4}{D_2}^2 = \frac{\pi}{4} \times 0.10^2 = 0.00785\ m^2$$

From continuity equation we have,

$A_1V_1 = A_2V_2 = Q$

$$\therefore V_1 = \frac{Q}{A_1} = \frac{0.03}{0.01767} = 1.697\frac{m}{s} \qquad V_2 = \frac{Q}{A_2} = \frac{0.03}{0.00785} = 3.82\ m/s$$

Applying Bernoulli's Equation before and after contraction,

$$\frac{P_1}{\rho g} + \frac{V_1}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2}{2g} + Z_2 + h_f$$

But $Z_1 = Z_2$

Head loss due to contraction is given by,

$$h_c = \frac{{V_2}^2}{2g}\left[\frac{1}{C_c} - 1\right]^2 = \frac{3.82^2}{2 \times 9.81}\left[\frac{1}{0.6} - 1\right]^2 = 0.33$$

Substituting the value in Bernoulli's equation,

$$\frac{P_1}{\rho g} + \frac{V_1}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2}{2g} + Z_2 + 0.33 \qquad \therefore \quad P_1 - P_2 = 0.909\ N/Cm^2$$

Pressure loss across the contraction= $0.909\ N/Cm^2$

2) *Show that the two dimensional flow described by the eq* $\varphi = x + 2x^2 - 2y^2$ *Is irrotational. What is the velocity potential of the flow? If the density of the fluid is 1.12 Kg/m*^{3} *and the pressure at a point (1,-2) is 4.8 KPa what is the pressure at point (9, 6)?*

$\varphi = x + 2x^2 - 2y^2$

Laplace Eq^n *gives,*

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} = 0 \therefore \frac{\partial^2(x + 2x^2 - 2y^2)}{\partial x^2} + \frac{\partial^2(x + 2x^2 - 2y^2)}{\partial y^2}$$

$$\frac{\partial^2(x + 2x^2 - 2y^2)}{\partial x^2} + \frac{\partial^2(x + 2x^2 - 2y^2)}{\partial y^2} \therefore \frac{\partial}{\partial x}(1 + 4x) + \frac{\partial}{\partial y}(-4y)$$

=4+(-4)

= 0

Laplace equation satisfies therefore the flow is irrotational.

Now Velocity potential of the flow= $(\emptyset)$

$$\frac{\partial\emptyset}{\partial x} = \frac{\partial\varphi}{\partial y} \therefore \frac{\partial\emptyset}{\partial x} = \frac{\partial}{\partial y}(x + 2x^2 - 2y^2) \therefore \frac{\partial\emptyset}{\partial x} = (-4y) \therefore \emptyset = -4yx + c$$

$$\frac{\partial\emptyset}{\partial y} = -4x + \frac{\partial c}{\partial y} \quad but\frac{\partial\emptyset}{\partial y} = \frac{-\partial\varphi}{\partial y} = -(1 + 4x)$$

$$-(1 + 4x) = -4x + \frac{\partial c}{\partial y} \quad \therefore -1 = \frac{\partial c}{\partial y} \therefore c = -y \therefore \emptyset = -4yx - y$$

Given, Density= 1.12 Kg/m^3 *(x, y) = (1, 2) P=4.8 KPa*

$$V_1 at\ (1,2) = \sqrt{u^2 + v^2} = 9.43\frac{m}{s} \quad V_2 at\ (9,6) = \sqrt{u^2 + v^2} = 44.1\frac{m}{s}$$

$$\frac{P_2}{\rho g} = \frac{V_1^2}{2g} - \frac{V_2^2}{2g} + \frac{P_2}{\rho g} \qquad \therefore P_2 = 4.78kpa$$

3) *Given Data,*

Tube diameter= 1mm

Surface tension coefficient= 0.0712

We know that,

$$h = \frac{2\sigma cos\theta}{\rho gr} \therefore h = \frac{2 \times 0.0712 \times cos0}{1000 \times 9.81 \times \frac{1}{2}/1000} \therefore h = \frac{0.1424}{4.915} \therefore h = 0.02m\ or\ 2\ Cm$$

True Water Height:

$True\ Water\ Height = H - h \therefore H_T = 100 - 20 = 80\ mm$

Q.2. 1*) the stream function for combined flow is given by,*

$\varphi = K_1\theta_1 - K_2\theta_2 \ = \frac{\theta_1}{2\pi}\theta_1 - \frac{\theta_2}{2\pi}\theta_2$

Considering the point P, $\theta_2 = \frac{\pi}{2}$ *and* θ_1 *is calculated*

$$tan\theta_1 = \frac{1}{2} = 0.5 \ \therefore \theta_1 = 26.6^0$$

Substituting the value of $\theta_1 and \theta_2$ *in above equation we get,*

$$\varphi_p = \frac{4}{2\pi} \times 0.46 - \frac{8}{2\pi} \times \frac{\pi}{2}$$

The general form of the stream function in rectangular co-ordinates can be written as,

$u = \frac{Q_1}{2\pi}\left[\frac{x+a}{x+a^2+y^2}\right] - \frac{Q_2}{2\pi}\left[\frac{x-a}{x+a^2+y^2}\right] \quad v = \frac{Q_1}{2\pi}\left[\frac{y}{x+a^2+y^2}\right] - \frac{Q_2}{2\pi}\left[\frac{y}{x+a^2+y^2}\right]$

Here given data,

$\theta_1 = 3\frac{m^3}{s} \quad \theta_1 = 4\ \frac{m^3}{s} \quad a = 2\ and\ at\ p\ x = 0, \quad y = 2$

$u = \frac{3}{2\pi}\left[\frac{0+2}{2^2+2^2}\right] - \frac{4}{2\pi}\left[\frac{0-2}{-2^2-2^2}\right] = \frac{3}{2\pi}\left[\frac{2}{8}\right] - \frac{4}{2\pi}\left[\frac{-2}{8}\right] = \frac{1}{2\pi}\left[\frac{14}{8}\right] = \frac{4}{2\pi}\left[\frac{14}{8}\right] = 0.27\ m/s$

$v = \frac{3}{2\pi}\left[\frac{2}{2^2+2^2}\right] - \frac{4}{2\pi}\left[\frac{2}{-2^2+2^2}\right] = \frac{3}{2\pi}\left[\frac{2}{8}\right] - \frac{4}{2\pi}\left[\frac{2}{8}\right] = \frac{1}{2\pi}\left[\frac{2}{8} - \frac{2}{8}\right] = 0.00\ m/s$

Resultant Velocity

$V = \sqrt{u^2 + v^2} = \sqrt{0.27^2 + 0^2} = 0.27\ m/s$

2) *Given Data,*

$R = \frac{D}{2} = \frac{1.2}{2} = 0.6\ m$ *Length of tank= l = 1.2 m Pressure P= 14 Kpa*

$\therefore Pressure\ Head\ \ h = \frac{P}{\rho g} = \frac{1.4 \times 10^4}{1000 \times 9.81} \therefore h = \frac{14000}{9810} \ \therefore h = 1.42\ m$

(i) *Horizontal component of Force,*

$F_x = \rho g A\bar{h}$

Where A= area projected on vertical plane=0.6x1.2=0.72m^2

$\bar{h} = 1.42 + \frac{0.60}{2} = 1.42 + 0.3 = 1.72\ m$

$F_X = 100 \times 9.81 \times 0.72 \times 1.72 = 12148.7\ N$

(ii) *Vertical Component of Force*

F_Y= *Weight of the water enclosed or supported actually or imaginary by curved surface ABC*

=Weight of the water in portion CODEABC

=Weight of the water in CODFBC-Weight of the water in AEFB

But Weight of the water in CODFBC= Weight of the water in (COB+ODFBO)

$\rho g\left[\frac{\pi r^2}{4} + BO + OD\right] \times 2 = 1000 \times 9.81\left[\frac{\pi 0.6^2}{4} + 0.6 + 1.42\right] \times 2 = 17226.36\ N$

Weight of the water in AEFB= $\rho g[Area\ of\ AEFB] \times 2 = 1759.91\ N$

$\therefore\ F_y = 17226.36 - 1759.91 = 15466.45\ N$

$Q.3.1)$

Given data,

Length of pipe1 = L_1 =100 m

Length of pipe2 = L_2 =100 m

Total rate of flow = Q = 2000 lit/min = 2 m^3/s

Co-efficient of Friction = $f_1=f_2=0.02$

Let Q_1, Q_2, *are rate of flows in pipe 1 and 2 respectively* $Q = Q_1 + Q_2$ *but* Q= 2 Q_2

$3Q_2=2$ $Q_2=0.66\ m^3/s$ $2= Q_1+0.66= 1.34\ m^3/s$

$Q_1=1.34\ m^3/s$ $Q_2=0.66\ m^3/s$

$$h_f = \frac{4f_1L_1V_1^2}{D_1 \times 2g} = \frac{4f_2L_2V_1^2}{D_2 \times 2g}$$

The equation reduces to,

$$\frac{V_1^2}{D_1} = \frac{V_2^2}{D_2}$$

$Now\ Q_1 = \frac{\pi}{4} D_1{}^2 \times V_1$

Applying Bernoulli's equation to a and b

$$\frac{P_a}{\rho g}+\frac{V_a}{2g}+z_1-\frac{P_b}{\rho g}+\frac{V_b}{2g}+z_b+h_f$$

Q.4. 1) Given,

$$\frac{U}{U_\infty}=\left(\frac{y}{\delta}\right)^{1/7}$$

Substituting the value in Von Karman integral Equation,

$$\frac{\tau_0}{\rho U^2}=\frac{\partial}{\partial x}\left[\int_0^{\delta}\left[\frac{y}{\delta}\right]^{1/7}\left[1-\left(\frac{y}{\delta}\right)^{1/7}\right]dy\right]=\frac{\partial}{\partial x}\left[\int_0^{\delta}\left[\frac{y}{\delta}\right]^{1/7}-\left(\frac{y}{\delta}\right)^{1/7}\right]dy\right]$$

$$\frac{\tau_0}{\rho U^2}=\frac{\partial}{\partial x}\left[\frac{y^{1.7+1}}{\left(\frac{1}{7}+1\right)\delta^{1/7}}-\frac{y^{2.7+1}}{\left(\frac{2}{7}+1\right)\delta^{2/7}}\right]$$

By solving this we get,

$$\frac{\tau_0}{\rho U^2}=\frac{7}{72}\frac{\partial\delta}{\partial x}$$

Now, $\tau_0=\frac{7}{72}\rho U^2\frac{\partial\delta}{\partial x}$

But the value of τ_0 *for turbulent boundary layer is given,*

$$\tau_0=0.0225\rho U^2\left[\frac{\mu}{\rho U\delta}\right]^{1/4}$$

Equating the two values,

$$\frac{7}{72}\rho U^2\frac{\partial\delta}{\partial x}=0.0225\rho U^2\left[\frac{\mu}{\rho U\delta}\right]^{1/4}$$

By solving this we get, $=0.2314\left(\frac{\mu}{\rho U}\right)^{1/4}\partial x$

Integrating we get,

$$\frac{\delta^{1/4+1}}{\frac{1}{4}+1}=0.2314\left(\frac{\mu}{\rho U}\right)^{1/4}\partial x+c\ \textit{Where c is the constant of integration}$$

We get $\delta=0.37\left[\left(\frac{\mu}{\rho U}\right)^{1/5}\right]x^{1/5}$

Substituting the values of delta in equation we get,

$$\tau_0 = 0.0577\frac{\rho U^2}{2}\left[\frac{\mu}{\rho U x}\right]^{1/5}$$

Drag Force F_D:

$$F_D = \int_0^L \tau_0 \times b \times dx = \int_0^l 0.0577\frac{\rho U^2}{2}\left[\frac{\mu}{\rho U x}\right]^{1/5}\frac{1}{x^{1/5}} b\,dx$$

By solving this we get,

$$F_D - 0.72\frac{\rho U^2}{2}\left[\frac{\mu}{\rho U}\right]^{1/5} b l^{4/5}$$

Drag CO-efficient C_D.

$$C_D = \frac{F_D}{\frac{1}{2}\rho A U^2} \qquad Where\ A = L \times b$$

$$C_D = \frac{0.72 \times \frac{\rho U^2}{2} \times \left[\frac{\mu}{\rho U}\right]^{1/5} \times b \times l^{4/5}}{\frac{1}{2}\rho A U^2}$$

By solving this we get,

$$C_D = \frac{0.072}{R_{eL}^{1/5}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots Ans$$

2) Given Data,

$\bar{h}$= *Distance of C.G. from free liquid surface =x+0.61*

$\bar{\bar{h}}$= *Distance of C.P. from free liquid surface=x+0.686*

We know that,

$$\bar{\bar{h}} = \frac{I_G}{A\bar{h}} + \bar{h}$$

$$\therefore x + 0.686 = \frac{0.18}{1.48 \times [x + 0.61]} + [x + 0.61]$$

$$\therefore\ x + 0.686 = \frac{0.12}{x + 0.61} + x + 0.61$$

$$\therefore 0.076 = \frac{0.12}{x + 0.61}\ \therefore (x + 0.61) \times 0.076 = 0.12$$

By solving this we get

X =0.97 m

$\therefore \bar{h} = x + 0.61 = 0.97 + 0.61 = 1.58$

Force exerted on submerged surface

Total force= $\rho g A \bar{h} = 1000 \times 9.81 \times 1.48 \times 1.58 = 22.93\ KN \ldots\ldots\ldots\ldots.Ans.$

Q.5. 1) *Assumptions:*

(i) *The fluid is incompressible*

(ii) *The flow is one dimensional*

(iii) *The flow is steady*

(iv) *The flow is independent of any variation in z direction*

(v) *Body force per unit mass are zero*

$F_X=F_Y=F_Z=0$

From continuity equation,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

From Nervier stokes equation in x- direction,

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial u}{\partial t} = F_X - \frac{1}{\rho}\frac{\partial P}{\partial X} + \frac{\mu}{\rho}\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right]$$

$$\therefore 0 = -\frac{1}{\rho}\frac{\partial P}{\partial X} + \frac{\mu}{\rho}\left[\frac{\partial^2 u}{\partial y^2}\right]$$

We get $\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{\partial P}{\partial X}$ *Integrating we get*

$$\frac{\partial u}{\partial x} = \frac{1}{\mu}\frac{\partial P}{\partial X}y + c_1$$

Again integrating we get,

$$u = \frac{1}{\mu}\frac{\partial P}{\partial X}\frac{y^2}{\partial x}\frac{y^2}{2} + c_1 y + c_2$$

Where c1 and c2 are constant of integration

Boundary conditions are,

(i) At y=0 u=0 c2=0

(ii) At y=b u=v

$$\therefore U = \frac{1}{2\mu}\frac{\partial P}{\partial X}b^2 + c_1 b \;\therefore C_1 = \frac{U + \frac{1}{2\mu}\frac{\partial P}{\partial X}b^2}{b}$$

Substituting the values of C_1 and C_2

$$U = \frac{1}{2\mu}\frac{\partial P}{\partial X}y^2 + \frac{U + \frac{1}{2\mu}\frac{\partial P}{\partial X}b^2}{b}b$$

$$u = \frac{U}{b}y + \frac{1}{2\mu}\left(-\frac{\partial P}{\partial X}\right)(by - y^2) \ldots\ldots\ldots\ldots Ans$$

2) *Consider a fluid element of length dx, dy, dz in the direction of x, y, z. let u, v, w, are inlet velocity components*

Mass of the fluid entering the face ABCD,

$$= \rho \times velocity\ in\ X direction \times Area\ of\ ABCD = \rho u(dy \times dz)$$

Mass of the fluid leaving the face EFGH

,

$$= \rho u(dy \times dz) + \frac{\partial}{\partial X}\rho u(dy \times dz)dx$$

Gain of mass in X direction= Mass through ABCD-Mass through EFGH

$$= \rho u(dy \times dz) - \rho u(dy \times dz) + \frac{\partial}{\partial X}\rho u(dy \times dz)dx = -\frac{\partial}{\partial X}(\rho u)dxdydz$$

Similarly,

The net gain of mass in Y direction and Z direction are given as,

$$In\ y\ direction = -\frac{\partial}{\partial y}(\rho v)dxdydz,\ \ In\ z\ direction = -\frac{\partial}{\partial z}(\rho w)dxdydz$$

Net gain of mass is given as

$$= -\left[\frac{\partial}{\partial X}(\rho u)dxdydz + \frac{\partial}{\partial y}(\rho v)dxdydz + \frac{\partial}{\partial z}(\rho w)dxdydz\right]$$

$$= -\left[\frac{\partial}{\partial X}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right]dxdydz$$

Since the mass is neither created nor destroyed in the fluid element, the net increase of mass per unit time in the fluid element must be equal to the rate of increase of mass

Mass of the element=$\rho dxdydz$

Rate of increase of mass with time = $\frac{\partial}{\partial t}(\rho dxdydz)$

Equating the two expressions we get,

$$= -\left[\frac{\partial}{\partial X}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right] dxdydz = \frac{\partial}{\partial t}(\rho dxdydz)$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial X}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

If the fluid is incompressible,

$$\frac{\partial u}{\partial X} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots Ans.$$

Q.6. 1. *Boundary layer separation:*

When a solid body is immersed in a flowing fluid, a thin layer of fluid called the boundary layer is formed adjacent to the solid body. In this thin layer of the fluid the velocity varies from zero to free stream velocity in the direction normal to the solid body, the thickness of the boundary layer increases the fluid layer adjacent to solid surface has to do work against surface friction at the expense of its kinetic energy. This loss of kinetic energy is recovered from the immediate fluid layer in contact with the layer adjacent to solid surface through momentum exchange process. Thus the velocity of the layer goes on decreasing. Along the length of the solid body, at a certain point a stage may come when the boundary layer may not be able to keep sticking to the solid body. If it can not provide kinetic energy to overcome the resistance offered by the solid body, the boundary layer will be separated from the surface. This phenomenon is called as boundary layer separation.When a solid body is immersed in a flowing fluid, a thin layer of fluid called the boundary layer is formed adjacent to the solid body. In this thin layer of the fluid the velocity varies from zero to free stream velocity in the direction normal to the solid body, the thickness of the boundary layer increases the fluid layer adjacent to solid surface has to do work against surface friction at the expense of its kinetic energy. This loss of kinetic energy is recovered from the immediate fluid layer in contact with the layer adjacent to solid surface through momentum exchange process. Thus the velocity of the layer goes on decreasing. Along the length of the solid body, at a certain point a stage may come when the boundary layer may not be able to keep sticking to the solid body. If it can not provide kinetic energy to overcome the resistance offered by the solid body, the boundary layer will be separated from the surface. This phenomenon is called as boundary layer separation

2) Methods of preventing separation of boundary layer: when the boundary layer separates from the surface, a certain portion adjacent to the surface has a back flow and eddies are continuously *formed in this region and hence continuously loss of energy takes place. Thus the separation of boundary layer is undesirable*

The following are the methods of preventing the boundary layer separation

1. *Suction of the slow moving fluid by suction slot*
2. *Supplying additional energy from blower*
3. *Providing a bypass in slotted wing*
4. *Rotating boundary in direction of flow*
5. *Providing small divergence in a diffuser*
6. *Providing the guide blades*
7. *Providing the trip wire ring in the laminar region for the flow over a sphere*

Q.7. 1) Given Data,

P_a=1.2 Kg/cm^2

We know that,

$(Z_1 + h)S_1 + h_1 = hS_{Hg} + Z_2S_2 + h_2 = P_A + \rho_1 g h_1 = \rho_2 g h_2 + P_B$

$P_A + \rho_1 g h_1 - \rho_2 g h_2 + P_B - P_A + 0.4 \times 0.8 \times 1000 \times 9.81 - \rho_2 g h_2 + P_B$

$117.72\ KN + 3.139\ KN = P_B + 0.1 \times 9.81 \times 13.6 \times 10^3 + 9.81 \times 1000 \times 0.15$

$117.72\ KN + 3.139\ KN = P_B + 13.3416 \times 10^3 + 1.4715 \times 10^3$

$117.72\ KN + 3.139\ KN = P_B + 14.81\ KN$

$120.85\ KN = P_B + 14.81\ KN$

$P_B = 106.0459\ KN \ldots\ldots\ldots\ldots\ldots\ldots\ldots.Ans.$

2) Given Data,

Viscosity: $\mu = 0.9$

Discharge: $Q = 20\frac{Lit}{sec} = 20 \times 10^{-3} m^3/s$

Density: $\rho = 1.26$

Pressure: $P_1 = 590\ Kpa$

Velocity at inlet end,

$$V_1 = \frac{Q}{A} = \frac{20 \times 10^{-3} m^3/s}{(0.1)^2 \times \frac{\pi}{4}} = 2.546\ m/s$$

For laminar flow Reynolds number is, $R_e = \frac{\rho VD}{\mu} = \frac{1.26 \times 2.546 \times 0.1}{0.9} = 358.44$

As the Reynolds number is less than 2000 therefore the flow is Laminar.

For laminar flow,

$$U_{Mean} = \frac{V_{Max}}{2} = \frac{2.546}{2} 1.273\ m/s$$

Shear stress,

$$\tau = \mu \frac{\bar{U}}{R} = 0.9 \times \frac{1.237}{0.05} = 22.266\ N/m^2$$

$$But\ \tau = \frac{F}{A}\ therefore\ F = \tau \times A = 22.266 \times \frac{\pi}{4} 0.1^2 = 314.776\ KN \ldots\ldots Ans.$$

3) Convert the following

I. *-200 mm of Hg into Kpa*

$P = \rho g h$

$P = 13.6 \times 9.81 \times 10^3 \times 200$

$P = 26.68\ Kpa \ldots\ldots\ldots\ldots\ldots\ldots\ldots Ans.$

As atmosphere is at 101.3 Kpa Pressure is 74.62 Kpa bellow atmosphere

II. *125990 Pascal into meters of oil column*

$P = \rho g h$

$1.25990 \times 10^3 = 9.81 \times 0.8 \times 10^3 \times h$

$h = 16.05\ m\ of\ oil \ldots\ldots\ldots\ldots\ldots Ans.$