Paper solution

Class:-TE(EXC) **Year:- winter 2010**

Sem:- V **Subject:- Principles of control system**

Subject Incharge- Mahamuni Poonam N.

Q.1 a)

Relation between poles & system dynamic response:-

i. If both roots are real, unequal & negative:

   V_0 (t)= A + B e^{-at} +C e^{-bt}

   Response is purely exponential, system is over-damped.

ii. If both roots are real, equal & negative:

   V_0 (t)= A + B t e^{-at} +C e^{-bt}

   Response is purely exponential & fastest, system is critically-damped.

iii. If both roots are complex conjugate with negative real part:

   V_0 (t)= A + B e^{-at} Sin(bt+θ)

   Response is damped oscillatory, system is under-damped.

iv. If both roots are purely imaginary:

   V_0 (t)= A + B Sin(bt)

   Response is purely sinusoidal, system is un-damped.

b)

Sensitivity:- The change in a particular variable due to the parameter can be expressed as sensitivity. Sensivity of system parameter 'T' o the parameter 'K' is

$$\text{Sensitivity} = \frac{\%\text{ Change in T}}{\%\text{ Change in K}}$$

Sensitivity of close loop system is:-

$$T(s) = \frac{G(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$\frac{dT(s)}{dG(s)} = \frac{[1+G(s)H(s)][1]-[G(s)H(s)]}{[1+G(s)H(s)]^2}$$

$$= \frac{1}{[1+G(s)H(s)]^2}$$

$$S_G^T = \frac{G(s)}{T(s)}\frac{dT(s)}{dG(s)}$$

$$= \frac{G(s)}{\left[\frac{G(s)}{1+G(s)H(s)}\right]} \frac{1}{[1+G(s)H(s)]^2}$$

$$S_G^T = \frac{1}{1+G(s)H(s)}$$

Thus sensitivity of close loop system can be reduced by the factor $[\frac{1}{1+G(s)H(s)}]$

c)

Root locus	**Routh's Criterion**
Graphical method	Tabular or array method
Parameter of system is varied from 0 to infinity	No such parameter variation
Rules are decided to construct root locus	First two rows are taken directly from characteristic equation
If all roots are in left half then system is stable	If all terms in first column have same sign then system is stable
Rouths criterion is used	Useful in root locus to find intersection with imaginary axis
Gives range of values of K for stability	It also gives such range

d)

Advantages of nyquist plot:-

i. Provides absolute stability information.
ii. The stability of close loop system is determined from open loop T.F.
iii. Indicates relative stability giving values of GM & PM.
iv. Useful in analyzing conditionally stable system.
v. Information regarding frequency response can be obtained.
vi. Indicates reality, the manner in which system should be compensated to yield desired response.

Q.2 a) i)

$V_b=V_0$

ii)

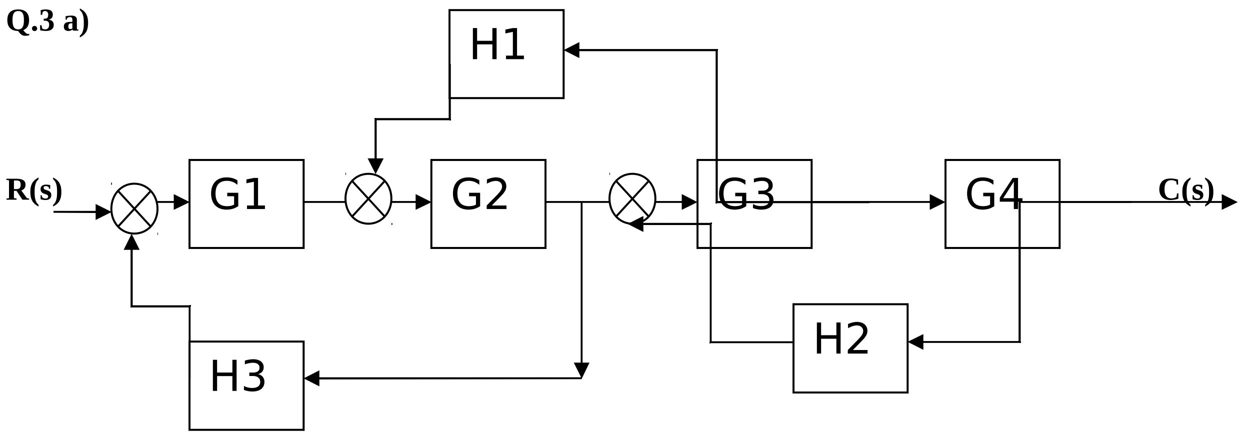
b) Effect of addition of zeros on 2nd order system:-

i. Peak overshoot occurs early in system.
ii. Magnitude of Peak overshoots increases.
iii. Smaller the value of z, the peaking phenomenon is dominant.
iv. Higher the value of z, the effect becomes more insignificant.

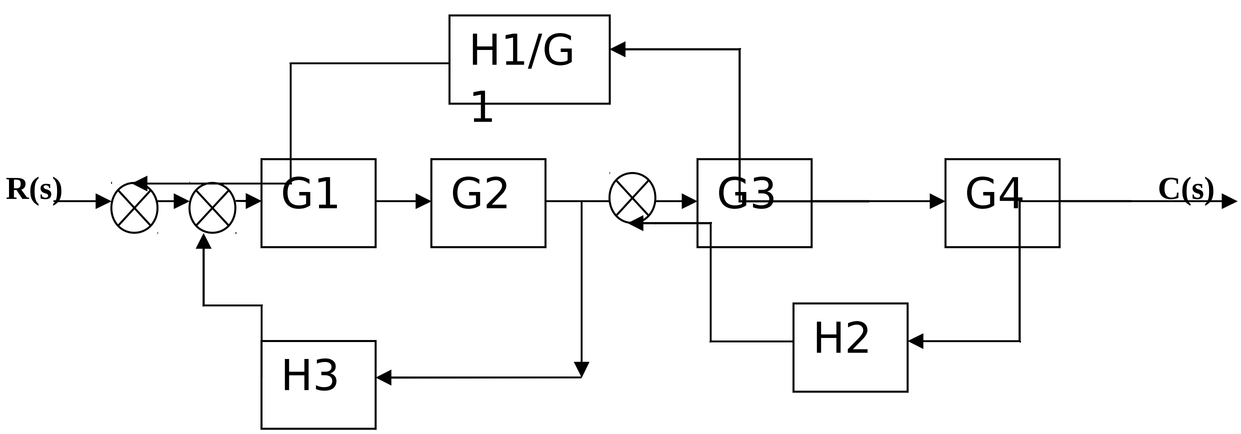
Effect of addition of poles on 2nd order system:-

i. Peak overshoot occurs late in system.
ii. System becomes more oscillatory in nature.
iii. Higher the value of p, the system is more unstable.
iv. Smaller the value of p, the effect becomes less insignificant.

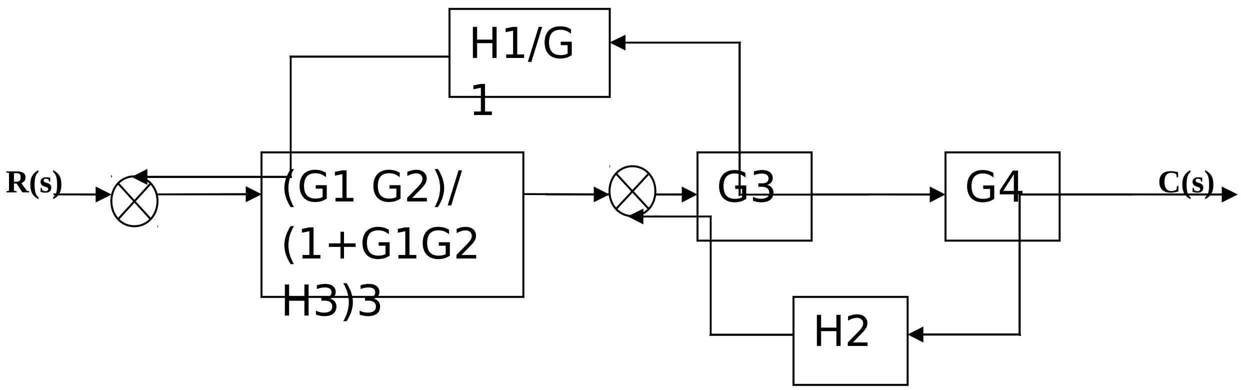
Q.3 a)



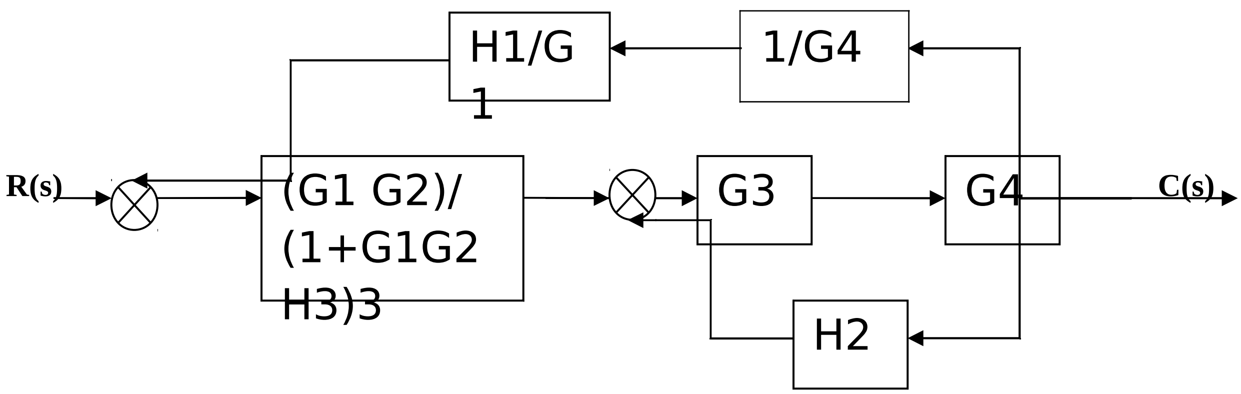
Shifting summing point before G1 block & using associated law.

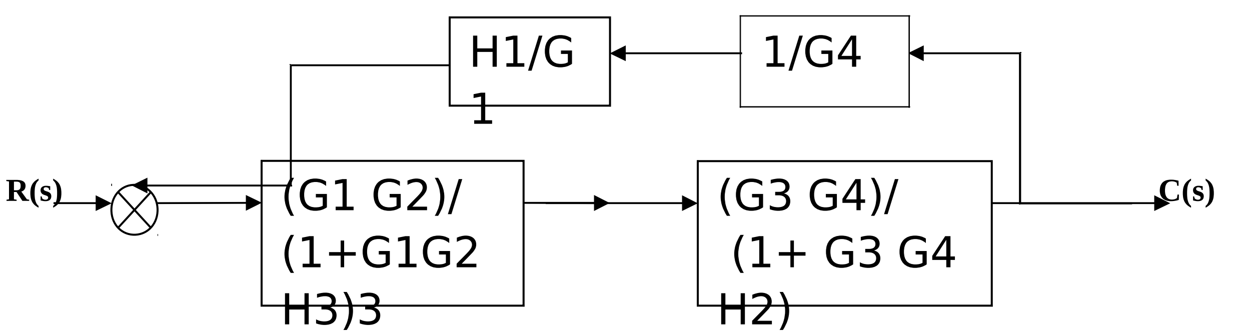


Reducing minor feedback loop.

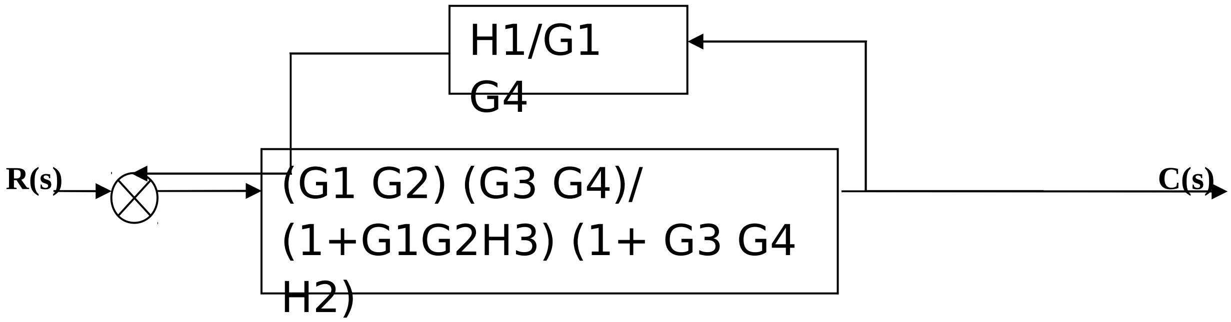


Shifting takeoff point after G4 block & reducing minor feedback loop.





Reducing blocks in series.



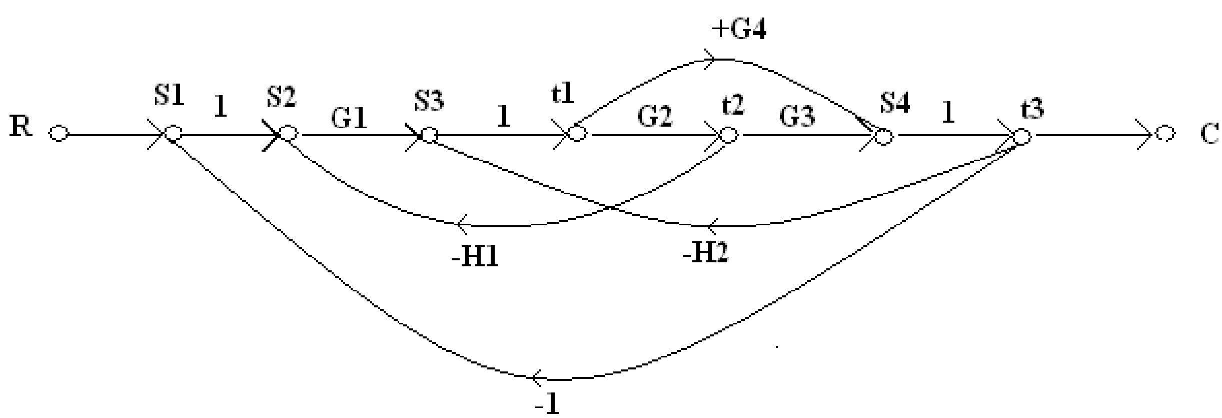
Reducing minor f/b loop.

Thus,

$$TF = \frac{R(s)}{C(s)} = \frac{G1\ G2G3\ G4}{1 + G1\ G2H3 + G3G4H2 + G2G3H1 + G1G2G3G4H2H3}$$

b)

SFG:-



No of forward paths=02

T1=G1G2G3

T2= G1 G4

Individual f/b loops=05

L1=- G1 G4
L2=- G1 G2 G3
L4=- G3 G2H2
L5=- G4 H2
No combination of non-touching loops.
All loops are touching to forward path.
Δ=1-[L1+L2+L4+L5]
For T1; we have
Δ1=1
For T2; we have
Δ2=1
Hus using masons gain formula,

$$\frac{C}{R} = \frac{T1\Delta1 + T2\Delta2}{\Delta}$$

$$\frac{C}{R} = \frac{G1\ G2G3 + G1G4}{1 + G1\ G4 + G1G2G3 + G1G2H1 + G2G3H2 + G4H2}$$

Q.4 a)
Response of 2nd order system to unit step input:-

b) Steady state error:-

i. Reference input is step of magnitude A:
Let,

$$R(s) = \frac{A}{s}$$

$$e_{ss} = \lim_{s\to 0} \frac{s\,R(s)}{[1 + G(s)H(s)]}$$

$$= \frac{s\,\frac{A}{s}}{1 + G(s)H(s)}$$

$$= \frac{A}{1 + \lim_{s\to 0}[G(s)H(s)]}$$

$$= \frac{A}{1 + K_p}$$

Where, $K_p = \lim_{s\to 0}[G(s)H(s)]$

ii. Reference input is ramp of magnitude A:
Let,

$$R(s) = \frac{A}{s^2}$$

$$e_{ss} = \lim_{s\to 0} \frac{s\,R(s)}{[1 + G(s)H(s)]}$$

$$= \frac{s\,\frac{A}{s^2}}{1 + G(s)H(s)}$$

$$= \frac{A}{s + \lim_{s\to 0}[s\,G(s)H(s)]}$$

$$= \frac{A}{K_v}$$

Where, $K_v = \lim_{s\to 0}[sG(s)H(s)]$

iii. Reference input is parabolic of magnitude A:

Let,

$$R(s) = \frac{A}{s^3}$$

$$e_{ss} = \lim_{s\to 0} \frac{s\,R(s)}{[1 + G(s)H(s)]}$$

$$= \frac{s\,\frac{A}{s^3}}{1 + G(s)H(s)}$$

$$= \frac{A}{s^2 + \lim_{s\to 0}[s^2\,G(s)H(s)]}$$

$$= \frac{A}{K_a}$$

Where, $K_a = \lim_{s\to 0}[s^2G(s)H(s)]$

Relation between steady state error & type of the system:-

Type of the system	Error coefficients			Error e_{ss} for		
	K_p	K_v	K_a	Step i/p	Ramp i/p	Parabolic i/p
0	K	0	0	$\frac{A}{K+1}$	∞	∞
1	∞	K	0	0	$\frac{A}{K}$	∞
2	∞	∞	K	0	0	$\frac{A}{K}$

Q.5 a)

i. Routh's stability Criterion:-

$s^4 + 9s^3 + 26s^2 + 24s + K = 0$

s^4	1	26	K
s^3	9	24	0
s^2	23.33	k	
s^1	(560-9k)/23.3		
s^0	k		

Marginal value of K makes row of s^1 as row of zero,

Thus,

$$\frac{560 - 9K}{23.33} > 0$$

Therefore,

$560-9K_{mar}=0$

$K_{mar}=62.22$

$A(s)=23.33\ S^2 + K=0$

$S^2 = -2.6$

$S=\pm j1.63$

Frequency of oscillation= 1.63 rad/sec.

ii. $s^5 + 6s^4 + 15s^3 + 30s^2 + 44s + 24 = 0$

s^5	1	15	44
s^4	6	30	24
s^3	10	40	0
s^2	6	24	
s^1	0	0	
s^0			

$A(s)=6S^2 + 24=0$

$$\frac{dA(s)}{ds} = 12s$$

Thus, array becomes

s^5	1	15	44
s^4	6	30	24
s^3	10	40	0
s^2	6	24	
s^1	12	0	
s^0	24		

There is no sign change thus no roots in RHS. The dominant roots are;

$A(s)=6S^2 + 24=0$

$6S^2 = 24$

$S^2 = -4$

$S=\pm j2$

Therefore two roots are on imaginary axis.

b) $G(s)H(s) = \frac{K}{s(s+2)(s+4)}$

Phase margin=60^0

Gain margin=20db

i. Pole at origin=1/s
ii. Simple pole=1/(s+2)
    T_1=1/2,ω_{c1}=2
iii. Simple pole=1/(s+4)
iv. T_2=1/4,ω_{c2}=4
v. Phase angle

$$G(j\omega)H(j\omega) = \frac{K'}{(j\omega)(j\omega+2)(j\omega+4)}$$

ω	1/jω	$-\tan^{-1}(j\omega/2)$	$-\tan^{-1}(j\omega/4)$	Φ_R
0.2	-90^0	-5.71^0	-2.86^0	-98.57^0
2	-90^0	-45^0	-26.56^0	-161.56^0
4	-90^0	-63.43^0	-45^0	-198.43^0
10	-90^0	-78.69^0	-68.19^0	-236.88^0
∞	-90^0	-90^0	-90^0	-270^0

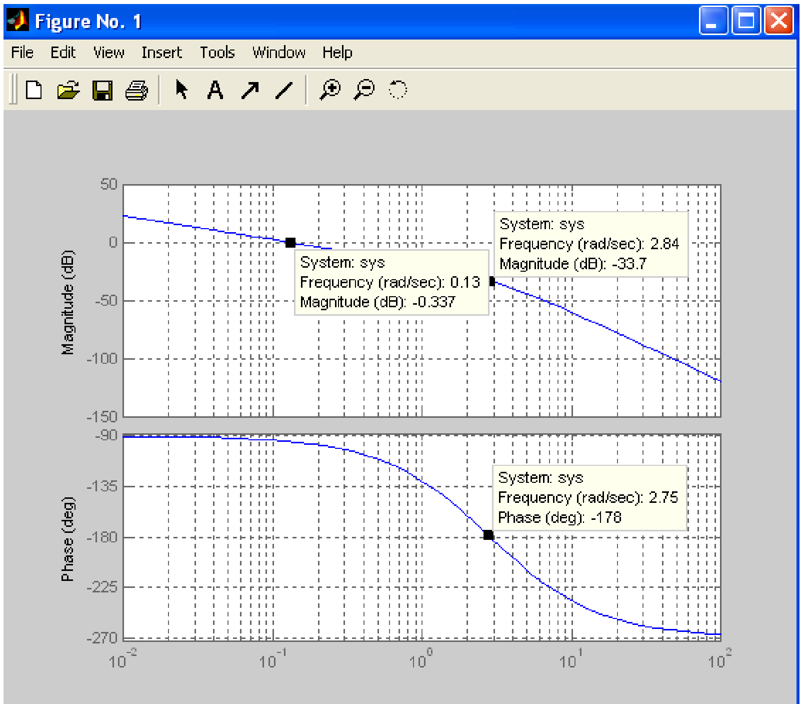
vi. Bode plot

From the plot,

20 logK'=-6

K'=K/8

K=4

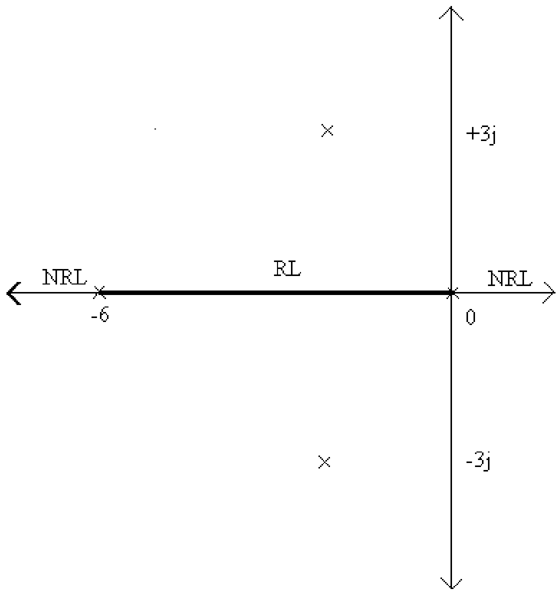


Q.6 a)

$$G(s)H(s) = \frac{K}{s(s+6)(s^2+4s+13)}$$

i. P=4, Z=0, N=4
   All branches are approaching to ∞
   Starting point of branches is s=0, -6, -2±3j

ii. Pole-zero plot



iii. Angle of Asymptotes

$$\theta = \frac{(2q+1)180^0}{P-Z}$$

Thus,

$\theta_1 = 45^0$

$\theta_2 = 135^0$

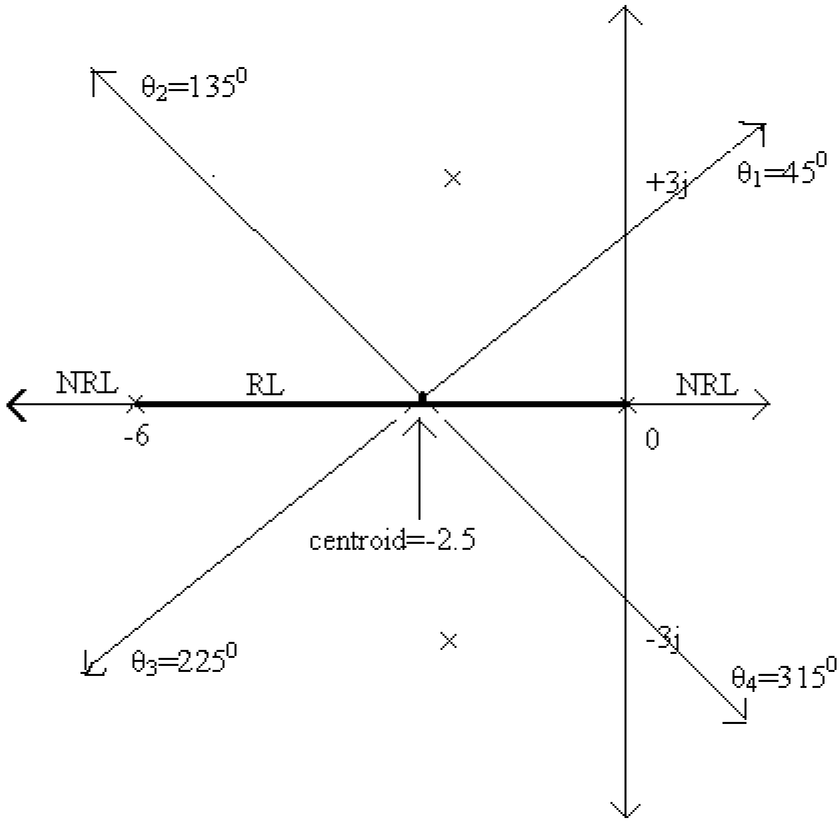
$\theta_3 = 225^0$

$\theta_4 = 315^0$

iv. Centroid

$$\sigma = \frac{\sum \text{R. P. of poles} - \sum \text{R. P. of zeros}}{P-Z}$$

$$\sigma = \frac{-6-2-2}{4} = -2.5$$



v. Breakaway points

Characteristic equation is: 1+G(s)H(s)=0

$$1+\frac{K}{s(s+6)(s^2+4s+13)}=0$$

$$s(s+6)(s^2+4s+13)+K=0$$

$s^4+10s^3+37s^2+78s+k=0$

$K=-s^4-10s^3-37s^2-78s$

$dK/ds=-4s^3-30s^2-74s-78=0$

$4s^3+30s^2+74s+78=0$

The roots of above equation are: -4.2, -1.649±1.3j

Lets check for valid breakaway point:

-4.2	4	30	74	78
		-16.8	-55.44	-77.95
	4	13.2	18.56	0.04=0

$$\frac{dK}{ds} = (s + 4.2)(4s2 + 13.2s + 18.56)$$

For s=-4.2
K=+104.63
Therefore s=-4.2 is a valid breakaway point.

vi. Intersection with imaginary axis
vii.
Characteristic equation is:

$s^4+10s^3+37s^2+78s+k=0$

S^4	1	37	K
S^3	10	78	0
S^2	29.2	K	
S^1	(2277.6-10K)/(29.2)	0	
S^0	K		

Therefore,

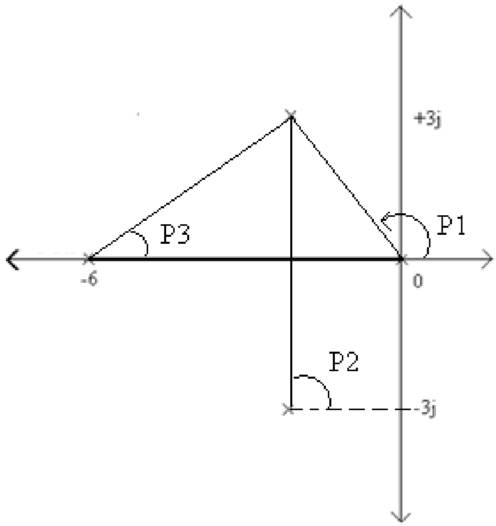
$K_{mar}=2277.6-10K_{mar}=0$
$K_{mar}=227.6$

$A(s)=29.2s^2+K=0$

$S=\pm 2.79j$

Thus root locus intersects with imaginary axis at $S=\pm 2.79j$

viii. Angle of departure



$\Phi_{p1}=180^0-56.3^0=123.69^0$

$\Phi_{p2}=90^0$

$\Phi_{p3}=36.86^0$

$\sum \Phi_p=250.55$ & $\sum \Phi_z=0$

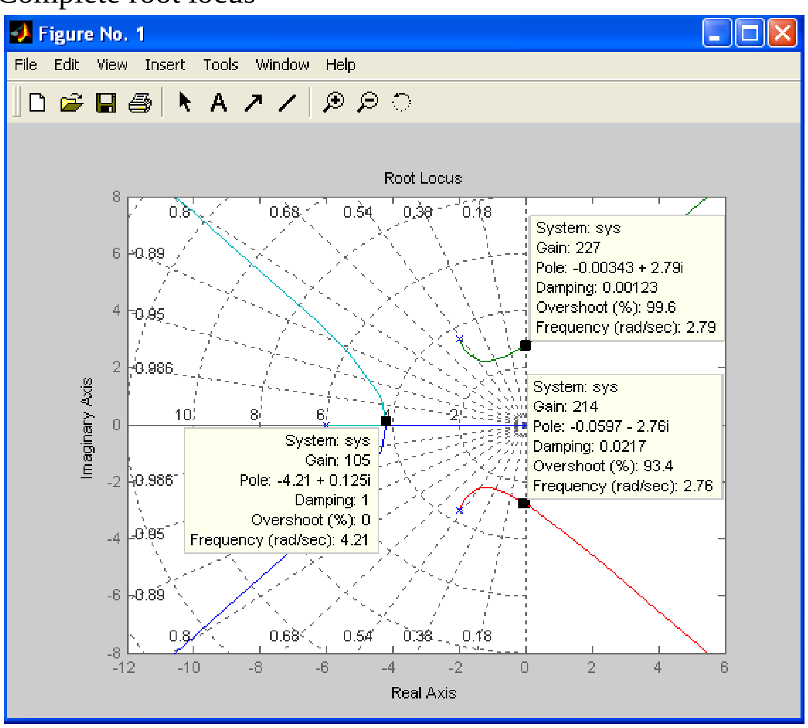
$\Phi_d=180-250.55$

Thus,

$\Phi_d= -70.55$ at $S=-2+3j$

$\Phi_d=70.55$ at $S=-2-3j$

ix. Complete root locus



x. Comment on stability
0 < K < 227.6 = System is Stable
K= 227.6 = Marginally stable
K > 227.6 = Unstable system

b)

$$G(s)H(s) = \frac{4}{s(s^2 + 16s + 4)}$$

$$G(s)H(s) = \frac{K}{s(s + 0.25)(s + 15.74)}$$

Time constant form is

$$G(j\omega)H(j\omega) = \frac{4}{j\omega(j\omega + 0.25)(j\omega + 15.74)}$$

$$G(j\omega)H(j\omega) = \frac{1.016}{j\omega\left(\frac{j\omega}{0.25} + 1\right)\left(\frac{j\omega}{15.74} + 1\right)}$$

i. K=1.016
20 logK=0.1378

ii. Pole at origin=1/s

iii. Simple pole= $\left(\frac{j\omega}{0.25} + 1\right)$

T_1=1/0.25 ω_{c1}=0.25

iv. Simple pole= $\left(\frac{j\omega}{15.74} + 1\right)$

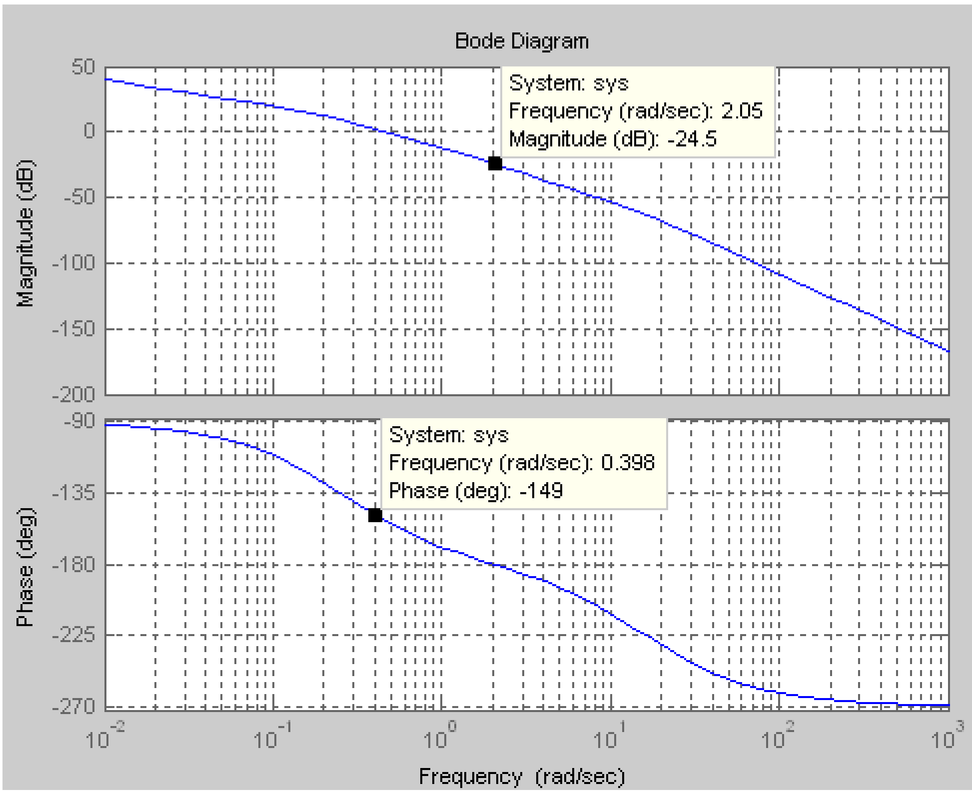
T_2=1/15.74 ω_{c1}=15.74

v. Phase angle

$$G(j\omega)H(j\omega) = \frac{1.016}{j\omega\left(\frac{j\omega}{0.25} + 1\right)\left(\frac{j\omega}{15.74} + 1\right)}$$

ω	1/jω	$-\tan^{-1}(j\omega/0.25)$	$-\tan^{-1}(j\omega/15.74)$	Φ_R
0.2	-90^0	-5.71^0	-2.86^0	-98.57^0
0.3	-90^0	-87.13^0	-17.621^0	-194.75^0
1	-90^0	-75.96^0	-3.63^0	-169.59^0
5	-90^0	-87.13^0	-17.62^0	-194.75^0
15	-90^0	-89.04^0	-43.62^0	-222.66^0
∞	-90^0	-90^0	-90^0	-270^0

vi. Bode plot



From the plot,

Gain margin= 24dB

Phase margin=30^0

Q.7

a) Error compensation methods & their effects

i. Static Error method:

Error coefficients are

$K_p = \lim_{s\to 0}[G(s)H(s)]$

$K_v = \lim_{s\to 0}[sG(s)H(s)]$

$K_a = \lim_{s\to 0}[s^2G(s)H(s)]$

Error associated is

$$e_{ss} = \frac{A}{1+K_p}$$

$$e_{ss} = \frac{A}{K_{vp}}$$

$$e_{ss} = \frac{A}{K_a}$$

ii. Dynamic Error method:

$$K_n = \lim_{s\to 0}\left(\frac{d^n F_1(s)}{ds}\right)^n$$

Where,

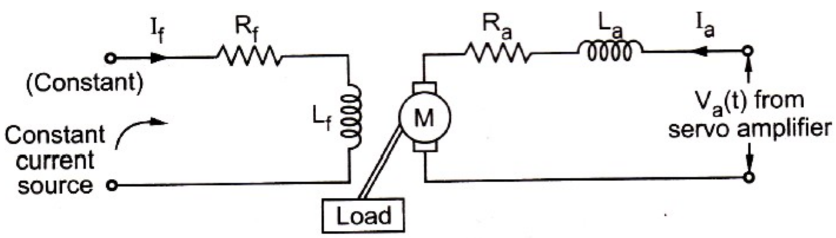
$$F_1(s) = \frac{1}{1+G(s)H(s)}$$

Advantages:-

i. Gives variations of error as a function of time.

ii. For any i/p other than standard i/p error can be determined.

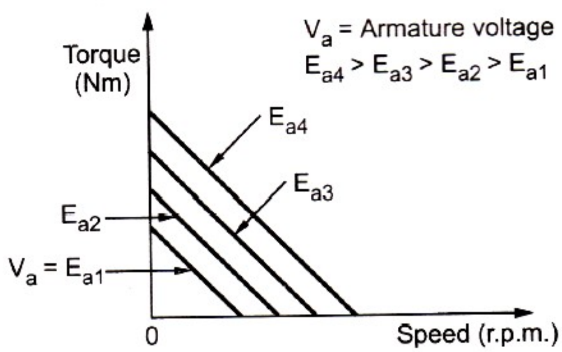
b) Armature Controlled D.C. Servomotor: In this type of motor, the input voltage 'v'" is applied to the armature with a resistance of Ra and inductance L". The field winding is supplied with constant current $I_{f.}$ Thus armature input voltage controls the motor shaft output. The arrangement is shown in the Fig.



Armature controlled d.c. servomotor

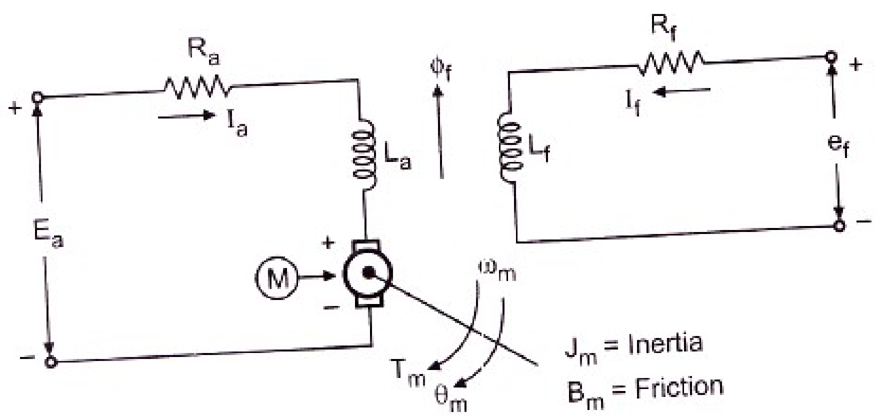
Features of Armature Gontrolled D'C' Servbmotor
It has following features :
i) Suitable for large rated motors.
ii) It has small time constant hence its response is fast to the control signal.
iii) It is closed loop system.
iv) The back e.m.f. provides internal damping which makes motor operation more stable.
v) The efficiency and overall performance is better than field controlled motor.

Characteristics of D.C. Servomotors



Torque-speed characteristics for an armature controlled d.c. servomotor

Transfer function:



1) Constant armature current is fed into the motor.

2) $\phi_f \propto I_f$. Flux produced is proportional to field current.

$$\therefore \quad \boxed{\phi_f = K_f I_f}$$

3) Torque is proportional to product of flux and armature current.

$$\boxed{T_m \propto \phi I_a}$$

$$\therefore \quad T_m = K' \phi I_a = K' K_f I_f I_a$$

$$\boxed{T_m = K_m K_f I_f} \qquad \text{... (1)}$$

where $K_m = K' I_a$ = constant

Apply Kirchhoff's law to field circuit.

$$L_f \frac{di_f}{dt} + R_f I_f = e_f \qquad \text{... (2)}$$

Now shaft torque T_m is used for driving load against the inertia and frictional torque.

$$T_m = J_m \frac{d^2\theta_m}{dt^2} + B_m \frac{d\theta_m}{dt} \qquad \text{... (3)}$$

$$\text{Inertia force} = J_m \frac{d^2\theta_m}{dt^2} \text{ similar to } m \frac{d^2x}{dt^2}$$

$$\text{Frictional force} = B_m \frac{d\theta_m}{dt} \text{ similar to } B \frac{dx}{dt}$$

Finding Laplace Transforms of equations (1), (2), and (3) we get,

$$T_m(s) = K_m K_f I_f(s) \qquad \text{... (4)}$$

$$E_f(s) = (sL_f + R_f)\ I_f(s) \qquad \text{... (5)}$$

$$T_m(s) = J_m s^2 \theta_m(s) + B_m s\, \theta_m(s) \qquad \text{... (6)}$$

Eliminate I_f (s) from equations (4) and (5)

$$T_m(s) = \frac{K_m K_f E_f(s)}{(sL_f + R_f)} \qquad \text{... (7)}$$

Eliminate $T_m(s)$ from equations (6) and (7),

$$(s^2 J_m + sB_m)\,\theta_m(s) = \frac{K_m K_f E_f(s)}{(sL_f + R_f)}$$

$$\text{Input} = E_f(s)$$

$$\text{Output} = \text{Rotational displacement } \theta_m(s)$$

$$\therefore \text{ Transfer function} = \frac{\theta_m(s)}{E_f(s)}$$

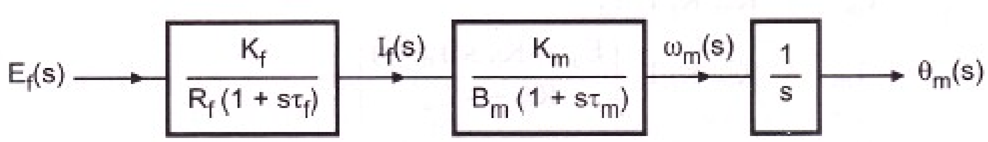
$$\frac{\theta_m(s)}{E_f(s)} = \frac{K_m K_f}{(J_m s^2 + sB_m)\ (R_f + sL_f)}$$

$$= \frac{K_m K_f}{sR_f\ B_m\ [1 + s\tau_m]\ [1 + s\tau_f]}$$

where $\tau_m = \dfrac{J_m}{B_m}$ = Motor time constant

$\tau_f = \dfrac{L_f}{R_f}$ = Field time constant

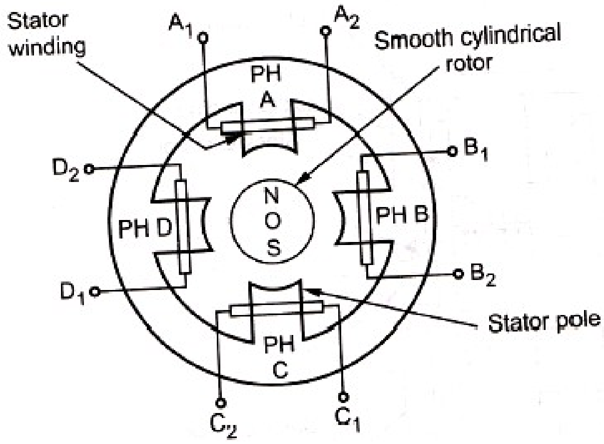
$$\text{T.F.} = \frac{\theta_m(s)}{E_f(s)} = \frac{K_f}{R_f[1 + s\tau_f]} \cdot \frac{K_m}{B_m(1 + s\tau_m)} \cdot \frac{1}{s}$$



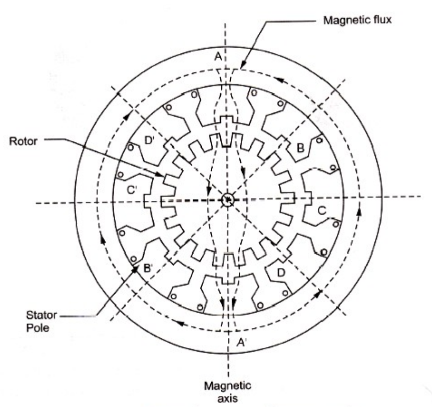
Block diagram

Q.7 c) Stepper motor construction:

1) Four phase permanent magnet stepper motor:

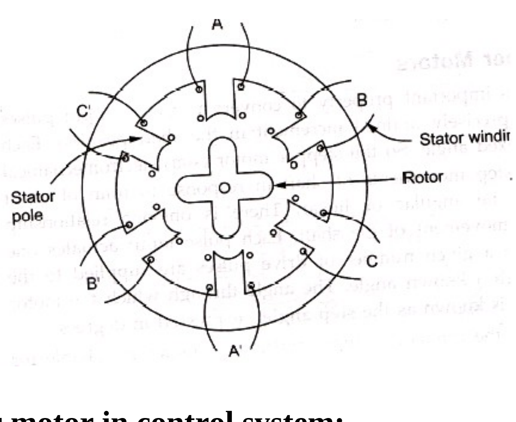


2) Reduction gear stepper motor:



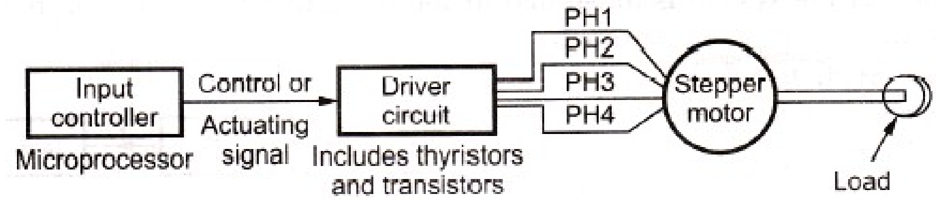
Reduction gear stepper motor

3) Variable reluctance motor.



Use of stepper motor in control system:-

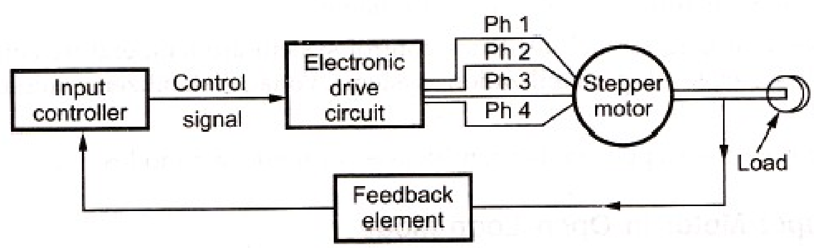
1) Stepper Motor in Open Loop Mode: We have seen that, motor speed is proportional to the rate of command pulses. Due to this property, it is well suited for open loop position and speed control applications. Generally stepper motors are operated by electronic circuits with a d.c. supply. Now a days, semiconductor devices such as thyristors and transistors are very common in the stepper motor driver circuits. In open loop mode, the controller which is some sort of integrated circuit or microprocessor generates a train of input pulses, as per the requirement. It not only generates the pulses but also controls the logic sequence, timings etc. of the pulses. Such a control signal generated is given to the driver circuit which accordingly excite the phases of the stepper motor. In turn the stepper motor controls the load to produce the necessary output. The block diagram of such a operation is shown in the Fig.



Stepper motor in open loop mode

Open loop control of the stepper motor is very simple and attractive' No feedback element like position or speed sensor is necessary. Since command is in pulses, the motor is compatible to digital systems. The open loop control is economical and has wide acceptance in speed and position control applications. But method has its own limitations' In some speed ranges/ the response of the stepper motor may become oscillatory' Similarly it fails to follow the input pulses when stepping rate is too high or the load inertia is too large. Due to these reasons the open loop mode of operation is limited.

2) Stepper Motor in Cosed Loop Mode: Due to the limitations of open loop control, a closed loop control of stepper motors is used in practice. In a closed loop control, the input controller gets the information about the ouput through the feedback element. Hence the driver circuit receives the control signal which is based on the feedback information. So switching of the motor takes place by means of train of input pulses, which is generated on the basis of feedback.



Stepper motor in closed loop mode

Close loop temperature control system is