

PAPER SOLUTION
SUB DIGITAL COMMUNICATION
EXAM WINTER 2010
CLASS T E EXTC
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Q 1a)

Inter channel interference in QPSK, a difficulty due to the wide spectrum of QPSK, is due to the nature of the base band signal. QPSK signal consists of abrupt changes and this changes give rise to spectral component of high frequencies. Filtering is not the right method to eliminate these problem. [Inter symbol interference will be the result]

In MSK, the baseband waveform that multiplies the quadrature carrier is much smoother than the abrupt rectangular baseband waveform of QPSK. The side lobes in MSK are relatively much smaller in comparison to the main lobe. So there is no inter-channel interference and inter symbol interference.

MSK waveform exhibits phase continuity i.e. there are no abrupt phase changes as in QPSK. So there is no inter symbol interference due to the nonlinear amplifiers in the system.

1 1b) The Signal to noise ratio is given by

$$\left[\frac{p_o^2(T)}{\sigma_o^2} \right]_{\max} = \frac{8E_s}{\eta}$$

We note that Eq establishes more generally the idea that the error probability depends only on the signal energy and not on the signal waveshape

1d)

∴ At transmitter

$$\begin{array}{l|l} d(t) = 1 & 1 \ 1 \ 0 \ 1 \ 1 \ 1 \\ b(t - T_b) = \underline{1} & 0 \ 1 \ 0 \ 0 \ 1 \ 0 \\ b(t) = 0 & 1 \ 0 \ 0 \ 1 \ 0 \ 1 \end{array}$$

Suppose this transmitted $b(t)$ now undergoes polarity inversion.

∴ At receiver

$$\begin{array}{l|l} d(t) = 0 & 0 \ 1 \ 1 \ 0 \ 1 \ 1 \\ b(t - T_b) = \underline{1} & 1 \ 0 \ 1 \ 1 \ 0 \ 1 \\ d(t) = 1 & 1 \ 1 \ 0 \ 1 \ 1 \ 1 \end{array}$$

same as transmitted $d(t)$

Hence, polarity inversion does not affect the performance of DPSK.

Q2 a) $I = \text{LOG}_2(1/P)$

$I_1 = .5146$, $I_2 = 2.7370$, $I_3 = 2.7370$

ENTROPY = $\sum p_k \cdot \log_2\left(\frac{1}{p_k}\right)$

$H = 1.18$

$S = rH = 4.725$

3a) Band limiting

Let $m(t)$ be a bandlimited signal with maximum frequency component f_m is periodically sampled every T_s seconds ($T_s \leq 1/2f_m$) using a train of impulses, then these samples $m(nT_s)$, $n = 0, \pm 1, \pm 2, \dots$, uniquely determine the signal $m(t)$ and the original signal can be reconstructed from $m(nT_s)$, using a low pass filter with a cut off frequency of f_m , without any distortion.

$$H_{LP}(f) = 1 \quad 0 \leq |f| \leq f_m$$

$$= 0 \quad \text{otherwise}$$

f_m = maximum modulating signal frequency, T_s = Sampling period,

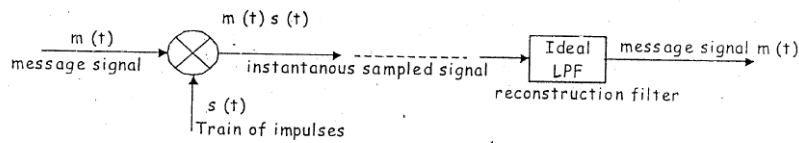
$$f_s = \frac{1}{T_s} = \text{Sampling frequency} \ \& \ f_s \geq 2f_m$$

$f_s = 2f_m$ is the minimum sampling frequency and is called Nyquist rate

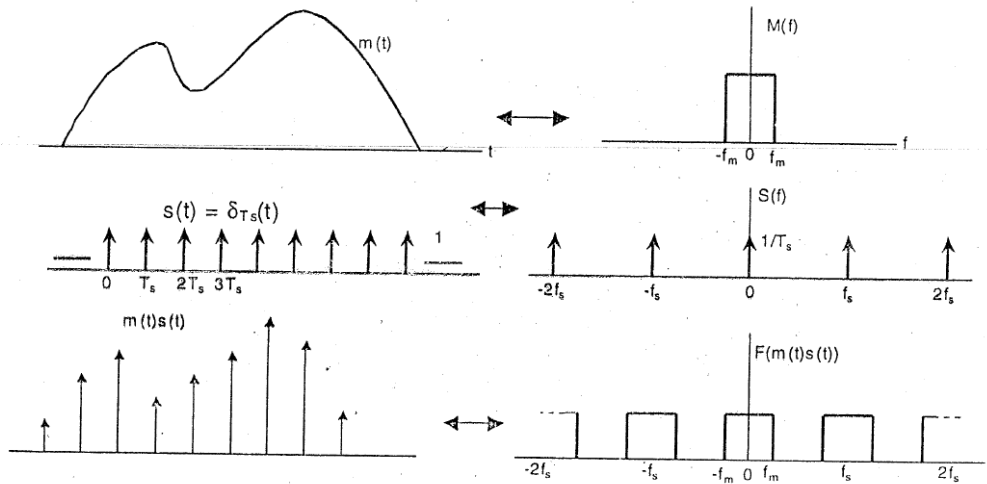
Sampling using train of impulses is called instantaneous sampling or impulse sampling or ideal sampling.

If $f_s > 2f_m$, then the sampling is over-sampling and $f_s < 2f_m$ then the sampling is under-sampling.

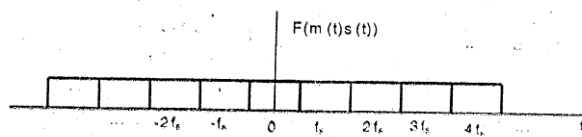
The sampling signal $\delta_{T_s}(t)$ is called ideal sampling function or Dirac comb.



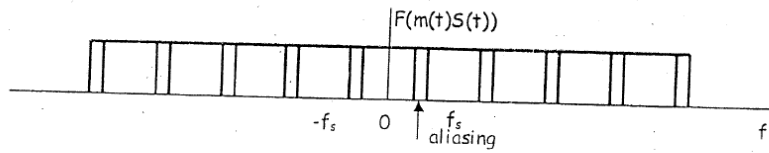
Proof Case -1 $f_s > 2f_m$



Case-2 $f_s = 2f_m$



Case-3 $f_s < 2f_m$



$$m(t) \leftrightarrow M(f)$$

$$s(t) \leftrightarrow \frac{1}{T_s} \sum_{n=-\infty}^{\infty} \delta(f - nf_s), \text{ where } s(t) \text{ is the sampling signal}$$

$$m(t) s(t) \leftrightarrow \left[M(f) * \frac{1}{T_s} \sum_{n=-\infty}^{\infty} \delta(f - nf_s) \right] = \frac{1}{T_s} \left[M(f) * \sum_{n=-\infty}^{\infty} \delta(f - nf_s) \right] = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} M(f - nf_s)$$

This is the F.T. of instantaneous sampled signals.

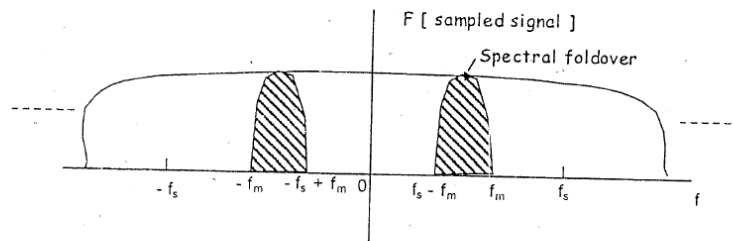
For avoiding distortion $f_s \geq 2f_m$

$$\text{i.e. } f_s \geq 2f_m \text{ or } T_s \leq \frac{1}{2f_m}$$

The spectrum overlap due to $f_s < 2f_m$ is called aliasing or foldover distortion or spectral folding. Due to aliasing unwanted frequency component will also be there with the reconstructed signal.

3.2 Aliasing or Fold over distortion

If f_s is less than $2f_m$ distortion will result. The distortion is called aliasing or fold over distortion.



When $f_s < 2f_m$, the side frequencies from one harmonic fold over in to the side band of another harmonic. The frequency that folds over is an alias of the input signal (hence, the name "aliasing or "fold over distortion"). If an alias side frequency from the first harmonic folds over into the audio spectrum, it cannot be removed through filtering or any other technique.

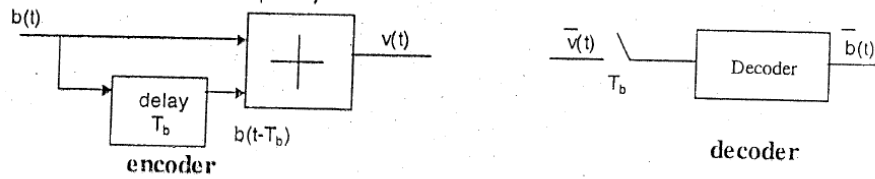
For avoiding aliasing $f_s > 2f_m$. The input filter (before sampling process) is called an anti-aliasing or pre-aliasing or anti-foldover filter. Its upper cut off frequency is chosen such that no frequency greater than one half of the sampling rate is allowed to enter the sample and hold circuit, thus, eliminating the possibility of fold over distortion occurring.

3 b)

3.15 Correlative Coding (Duo-binary Encoding)

In 1963, Adam Lender showed that it is possible to transmit R symbols / sec with zero ISI, using the theoretical minimum BW of $R/2$ Hz, without infinitely sharp filters. Lender used a technique called duobinary encoding or correlative coding. This is the practical method to implement Nyquist first criterion.

The basic idea behind the duobinary technique is to introduce some controlled amount of ISI in to the data stream rather than trying to eliminate it completely.



$$V(t) = b(t) + b(t - T_b)$$

	b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7
$b(t)$		0	0	1	0	1	1	0
Amplitudes	-1	-1	-1	1	-1	1	1	-1
$V(t)$		-2	-2	0	0	0	2	0
$\bar{V}(t)$		-2	-2	0	0	0	2	0
$\bar{b}(t)$		0	0	1	0	1	1	0

Let $b_0 = -1$ (or 1)

Decoding Rule

If $\bar{V}(t) = 2$, then $\bar{b}(t) = 1$

$\bar{V}(t) = -2$ then $\bar{b}(t) = 0$

$\bar{V}(t) = 0$ then opposite of the previous decision.

One drawback of this detection technique is that once an error is made it tends to propagate causing further error, since present decision depends on prior decision. A means of avoiding this error propagation is known as precoding.

Duobinary Encoder With Precoder

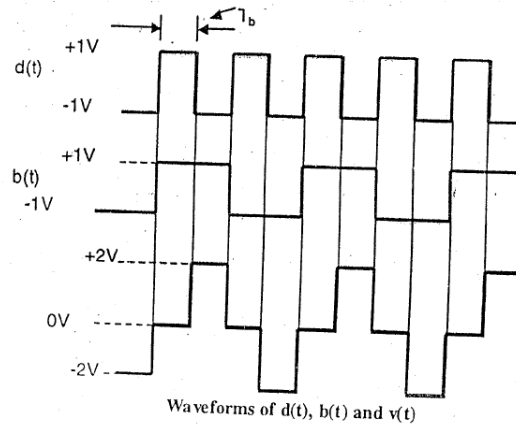
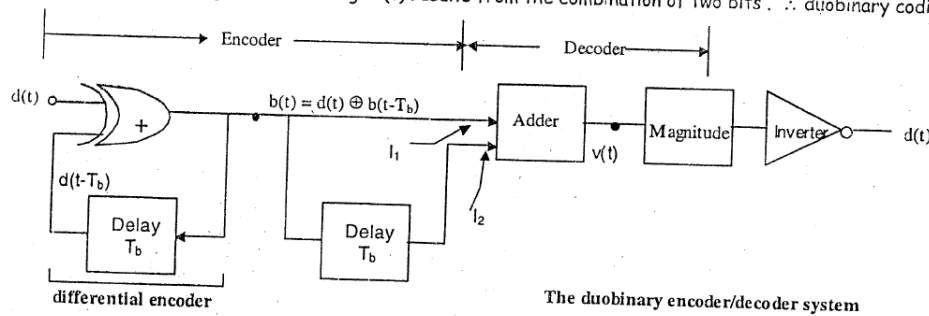
$$b(t) = d(t) \oplus b(t - T_b)$$

$$v(t) = b(t) + b(t - T_b)$$

The characteristic polynomial of duobinary encoder is $1 + D$

The value of $v(t)$ is in any interval k depends on both $b(t)$ and $b(t - T_b)$. Hence there is a correlation between the values of $v(t)$, in any two successive intervals \therefore correlative coding.

In each bit interval, the generated voltage $v(t)$ results from the combination of two bits \therefore duobinary coding.



I ₁		I ₂		v(t)	Magnitude	logic	$\bar{d}(t)$
Voltage	logic	Voltage	Logic				
-1	0	-1	0	-2	2	1	0
+1	1	-1	0	0	0	0	1
+1	1	+1	1	+2	2	1	0
-1	0	+1	1	0	0	0	1

From the above table $I_1 \oplus I_2 = d(t)$

but $I_1 = b(t) = d(t) \oplus b(t - T_b)$

$$I_2 = b(t - T_b)$$

$$\bar{d}(t) = I_1 \oplus I_2 = d(t) \oplus b(t - T_b) \oplus b(t - T_b)$$

$\therefore \bar{d}(t) = d(t)$, Thus hereby proved that output of the duobinary decoder is same as the input to the duobinary encoder.

Bandwidth

If the Baseband signal is $d(t)$, the required $BW = \frac{2}{T_s} = \frac{2}{2T_b} = f_b$

If the Baseband signal is $v(t)$, duobinary encoded signal, $BW = \frac{2}{T_s} = \frac{2}{4T_b} = \frac{f_b}{2}$

4.9 M-ary Phase Shift Keying System (MPSK)

N bits are lumped together \therefore period = NT_b , for the generation of a symbol

Total possible symbols = $2^N = M$ symbols

The M symbols are differ from one another by the phase $2\pi/M$

4 a)

$$V_{MPSK}(t) = \sqrt{2P_s} \cos(\omega_0 t + \phi_m) \text{ or } S_i(t) = \sqrt{2P_s} \cos\left(\omega_0 t + i \frac{2\pi}{M}\right), \text{ where } i = 0, 1, \dots, M-1$$

$$\text{Where } \phi_m = (2m+1) \frac{\pi}{M}, \text{ } m = 0, 1, 2, \dots, M-1$$

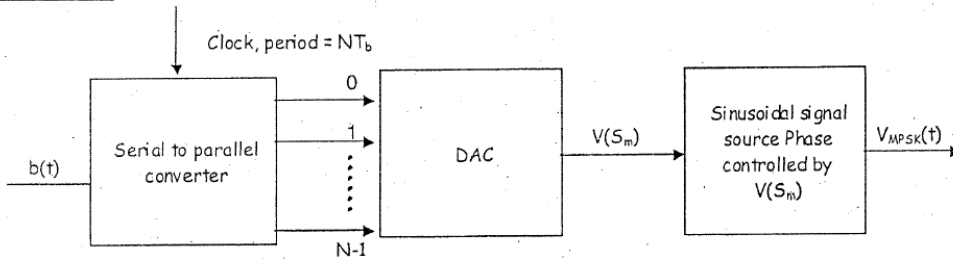
$$V_{MPSK}(t) = \sqrt{2P_s} \cos \phi_m \cos \omega_0 t - \sqrt{2P_s} \sin \phi_m \sin \omega_0 t$$

$$= P_e \cos \omega_0 t - P_o \sin \omega_0 t$$

$$\text{Where } P_e = \sqrt{2P_s} \cos \phi_m \text{ and } P_o = \sqrt{2P_s} \sin \phi_m$$

Both P_e and P_o can change every $T_s = NT_b$ sec and can assume any of M possible values.

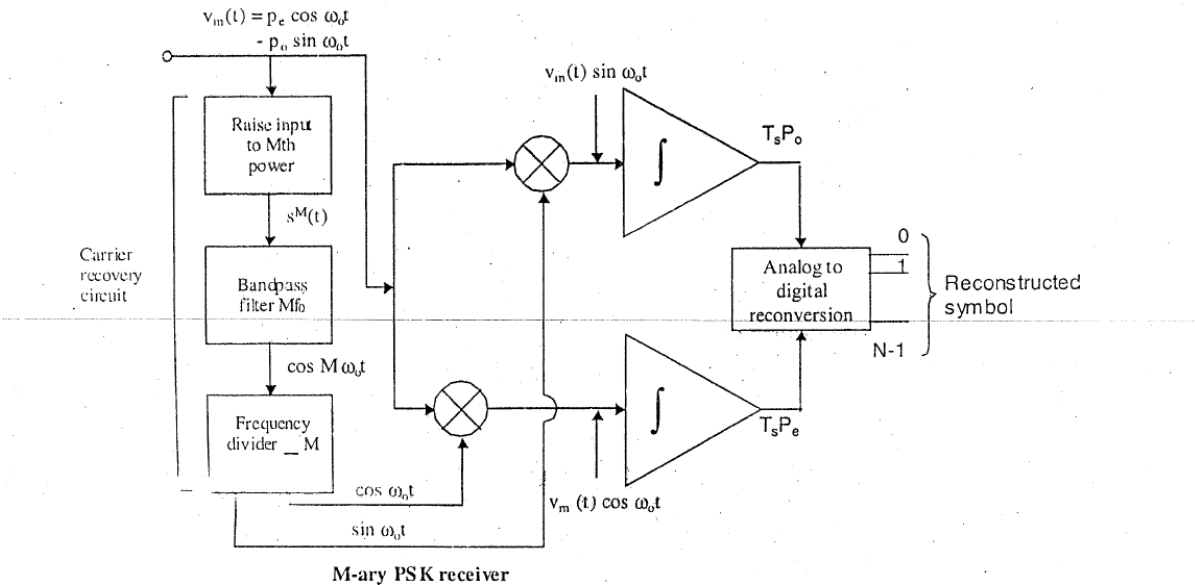
Transmitter



The converter has the facility for storing N bits of a symbol. Each symbol time the converter output is updated.

DAC output is a voltage $V(S_m)$ which depends on the symbol S_m ($m = 0, 1, \dots, M-1$). $V(S_m)$ is applied as a control input to a special type of constant amplitude sinusoidal signal source whose phase ϕ_m is determined by $V(S_m)$. The phase can change once per symbol time. (Symbol time $T_s = NT_b$)

Receiver



PSD Of MPSK Signal

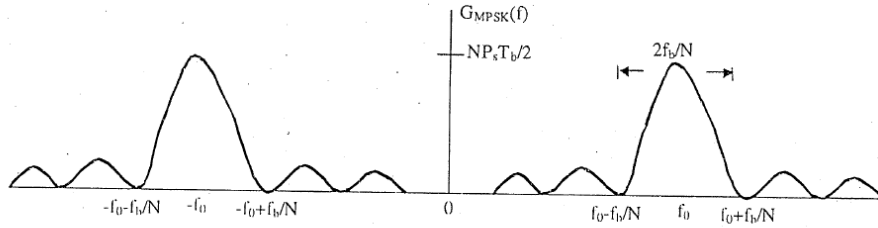
$$\text{PSD of the base signal, } G_B(f) = T_s S a^2 (\pi f T_s) \text{ where } T_s = NT_b$$

$$\text{PSD of the carrier signal, } G_C(f) = \left(\frac{P_s}{2}\right) [\delta(f - f_0) + \delta(f + f_0)]$$

$$G_{\text{MPSK}}(f) = G_B(f) * G_C(f)$$

$$G_{\text{MPSK}}(f) = \frac{P_s N T_b}{2} \left\{ \left[\text{Sa} \pi (f - f_0) N T_b \right]^2 + \left[\text{Sa} \pi (f + f_0) N T_b \right]^2 \right\}$$

$$\text{Null of null BW} = \frac{2f_b}{N} \text{ Hz} \quad \therefore N \uparrow M \uparrow \text{BW} \downarrow$$



$$\text{Spectral efficiency} = \frac{f_b}{2f_b} = \frac{N}{2} \text{ bits/s/Hz}$$

Geometrical Representation Of MPSK Signals

$$V_{\text{MPSK}}(t) = S_i(t) = \sqrt{2P_s} \cos \left(\omega_0 t + i \frac{2\pi}{M} \right) \quad i = 0, 1, \dots, M-1$$

$$\text{The reference signals, } R_i(t) = \sqrt{\frac{2}{T_s}} \cos \left(\omega_0 t + i \frac{2\pi}{M} \right)$$

$$S_i(t) = \sqrt{P_s T_s} \cdot R_i(t) = \sqrt{E_s} \cdot R_i(t)$$

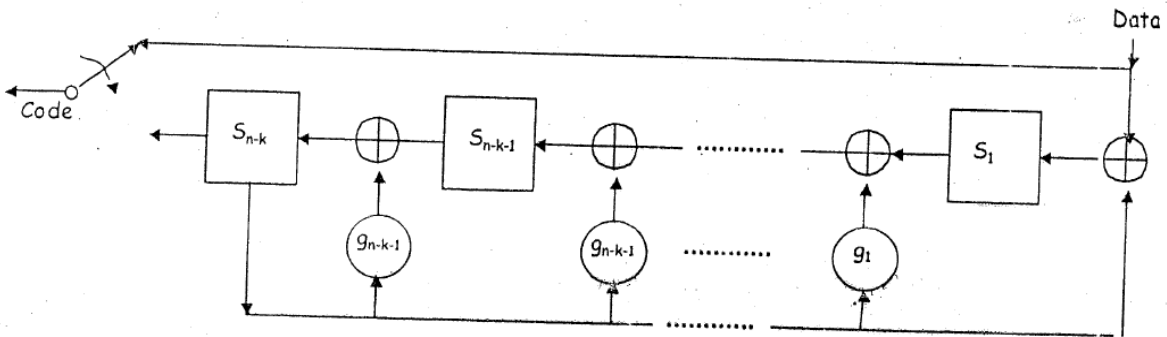
$$E_s + E_s - 2E_s \cos \left(\frac{2\pi}{M} \right) = d^2$$

$$\therefore d^2 = 2E_s \left\{ 1 - \cos \left(\frac{2\pi}{M} \right) \right\} = 2E_s \cdot 2 \sin^2 \left(\frac{\pi}{M} \right) = 4E_s \sin^2 \left(\frac{\pi}{M} \right)$$

$$\therefore d = \sqrt{4E_s \sin^2 \left(\frac{\pi}{M} \right)} = \sqrt{4NE_b \sin^2 \left(\frac{\pi}{2N} \right)} = 2\sqrt{E_s} \sin \left(\frac{\pi}{M} \right)$$

$N \uparrow d \downarrow$ except when $N=2$, for $N=1$ and $N=2$, same d .

Q 4b) encoder



CODE POLYNOMIAL

Systematic Cyclic code generation

$$C(x) = x^{n-k} D(x) \oplus m(x)$$

$\therefore C(x)$ = code polynomial of degree $n-1$

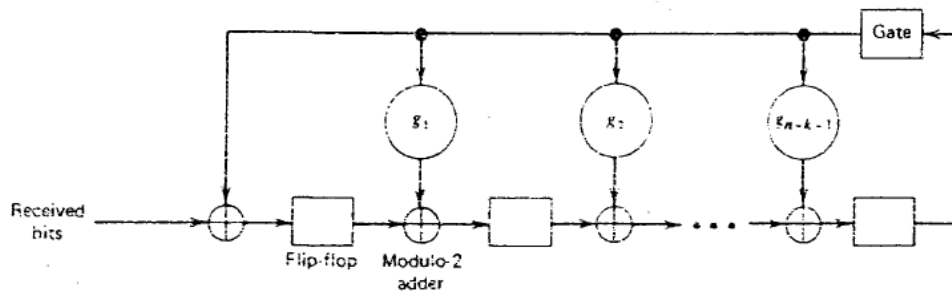
$D(x)$ = data polynomial of degree $k-1$

$m(x)$ = remainder of $\frac{x^{n-k} D(x)}{G(x)}$ and with degree $n-k-1$

$G(x)$ = Generator polynomial of degree $n-k$.

CODEWORD=[0 1 0 1 **1 0 0**]

Syndrome Calculator:



message	S2	S1	S0
1	0	0	0
1	0	0	1
0	0	1	1
1	1	1	0
1	1	1	0
0	1	1	0
0	1	1	1
	1	0	1

5 a) Bpsk waveform= $a \cos \omega t$ when $b(t)=1$

=- a coswtwhen b(t)=0

Dpsk waveform

d(t)	0	0	1	1	0	1	0	0	1	1	
b(t-1)	0	0	0	1	0	0	1	1	1	0	
b(t)	0	0	1	0	0	1	1	1	0		

$$v_{\text{DPSK}}(t) = b(t) \sqrt{2P_s} \cos \omega_0 t$$

$$= \pm \sqrt{2P_s} \cos \omega_0 t$$

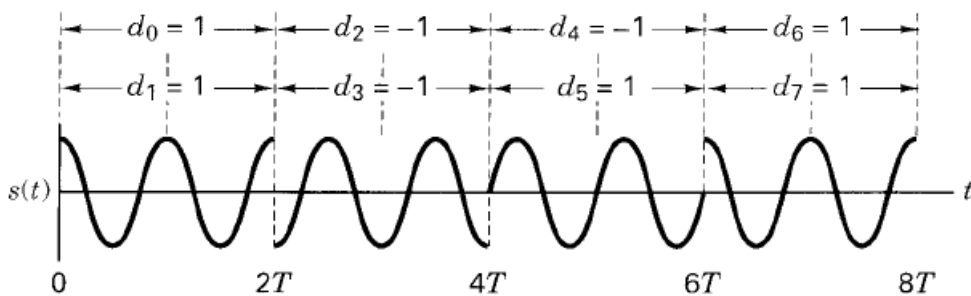
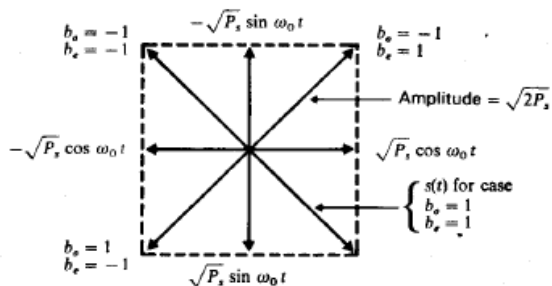
Fsk

fsk waveform= a cos2wtwhen b(t)=1

=- a coswtwhen b(t)=0

Qpsk

d(t)	0	0	1	1	0	1	0	0	1	1
odd	0	0	1	1	0	0	0	0	1	1
even		0	0	1	1	1	1	0	0	1



(c) QPSK

Que 5 b)

I) since $H = [P^T: I]$

$G =$

II) $C = M * G$

$$M = [0\ 0\ 1\ 1]$$

$$C = [0\ 0\ 1\ 1\ 1\ 1\ 0]$$

$$M = [1\ 0\ 0\ 1]$$

$$C = [1\ 0\ 0\ 1\ 1\ 0\ 0]$$

III) $S = 2$

$$T = 1$$

SINCE $D_{\text{MIN}} = 3$

1	1	1
1	1	0
1	0	1
0	1	1
1	0	0
0	1	0
0	0	1

$$\text{IV) } S = R * H^T = [1\ 0\ 0\ 0\ 1\ 1\ 0]^*$$

$$S = [0\ 0\ 1]$$

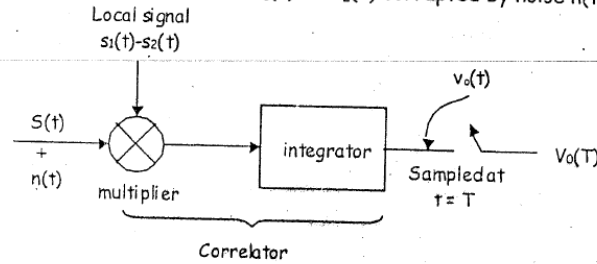
Error in last bit

Corrected code word = $[1\ 0\ 0\ 0\ 1\ 1\ 0]$; message = $[1\ 0\ 0\ 0]$

6 a)

3.23 Correlator Receiver (Correlation Realization of Matched filter)

Correlator is an alternate type of receiving system, which is identical in performance with the matched filter receiver. The input to the correlator is a binary data waveform $S_1(t)$ or $S_2(t)$ corrupted by noise $n(t)$. The bit length is T .



A coherent system of signal reception

There is a correlation between received signal and $S_1(t) - S_2(t)$. \therefore Correlator receiver. $S_1(t) - S_2(t)$ is generating from the received signal.

System	$S_1(t)$	$S_2(t)$	Local signal
Base band signal transmission	+V	-V	2V
BPSK	$A \cos \omega_0 t$	$-A \cos \omega_0 t$	$2A \cos \omega_0 t$
BASK	$A \cos \omega_0 t$	0	$A \cos \omega_0 t$
BFSK	$A \cos \omega_H t$	$A \cos \omega_L t$	$A \cos \omega_0 t - A \cos \omega_L t$

Signal & Noise Output Of Correlator

$$V_0(T) = \frac{1}{\tau} \int_0^T V_i(t) [S_1(t) - S_2(t)] dt,$$

Where $\tau = RC =$ time constant of integrator

$$V_0(T) = S_0(T) + n_0(T)$$

$$S_0(T) = S_{01}(T) \text{ or } S_{02}(T)$$

$$V_i(t) = S_i(t) + n(t)$$

$$S_i(t) = S_1(t) \text{ or } S_2(t)$$

$$\therefore S_0(T) = \frac{1}{\tau} \int_0^T S_i(t) [S_1(t) - S_2(t)] dt \quad \dots (C)$$

$$\therefore n_0(T) = \frac{1}{\tau} \int_0^T n(t) [S_1(t) - S_2(t)] dt \quad \dots (D)$$

Output of Matched filter (A&B) and output of correlator (C&D) are identical. Hence the performance of the two systems are identical.

\therefore Matched filter and correlator are two independent techniques of synthesizing the optimum filter $h(t)$

Examples of Correlator receiver

1. BPSK Receiver
2. QPSK Receiver
3. Coherent FSK Receiver
4. MSK Receiver

Maximum likelihood decision rule

The decoder must produce an estimate $\hat{\mathbf{u}}$ of the information sequence \mathbf{u} based on the received sequence \mathbf{r} . Equivalently, since there is a one-to-one correspondence between the information sequence \mathbf{u} and the code word \mathbf{v} , the decoder can produce an estimate $\hat{\mathbf{v}}$ of the code word \mathbf{v} . Clearly, $\hat{\mathbf{u}} = \mathbf{u}$ if and only if $\hat{\mathbf{v}} = \mathbf{v}$. A *decoding rule* is a strategy for choosing an estimated code word $\hat{\mathbf{v}}$ for each possible received sequence \mathbf{r} . If the code word \mathbf{v} was transmitted, a *decoding error* occurs if and only if $\hat{\mathbf{v}} \neq \mathbf{v}$. Given that \mathbf{r} is received, the *conditional error probability of the decoder* is defined as

$$P(E|\mathbf{r}) \triangleq P(\hat{\mathbf{v}} \neq \mathbf{v}|\mathbf{r}). \quad (1.6)$$

The *error probability of the decoder* is then given by

$$P(E) = \sum_{\mathbf{r}} P(E|\mathbf{r})P(\mathbf{r}). \quad (1.7)$$

$P(\mathbf{r})$ is independent of the decoding rule used since \mathbf{r} is produced prior to decoding. Hence, an optimum decoding rule [i.e., one that minimizes $P(E)$] must minimize $P(E|\mathbf{r}) = P(\hat{\mathbf{v}} \neq \mathbf{v}|\mathbf{r})$ for all \mathbf{r} . Since minimizing $P(\hat{\mathbf{v}} \neq \mathbf{v}|\mathbf{r})$ is equivalent to maximiz-

ing $P(\hat{\mathbf{v}} = \mathbf{v}|\mathbf{r})$, $P(E|\mathbf{r})$ is minimized for a given \mathbf{r} by choosing $\hat{\mathbf{v}}$ as the code word \mathbf{v} which maximizes

$$P(\mathbf{v}|\mathbf{r}) = \frac{P(\mathbf{r}|\mathbf{v})P(\mathbf{v})}{P(\mathbf{r})}, \quad (1.8)$$

that is, $\hat{\mathbf{v}}$ is chosen as the most likely code word given that \mathbf{r} is received. If all information sequences, and hence all code words, are equally likely [i.e., $P(\mathbf{v})$ is the same for all \mathbf{v}], maximizing (1.8) is equivalent to maximizing $P(\mathbf{r}|\mathbf{v})$. For a DMC

$$P(\mathbf{r}|\mathbf{v}) = \prod_i P(r_i|v_i), \quad (1.9)$$

since for a memoryless channel each received symbol depends only on the corresponding transmitted symbol. A decoder that chooses its estimate to maximize (1.9) is called a *maximum likelihood decoder* (MLD). Since $\log x$ is a monotone increasing function of x , maximizing (1.9) is equivalent to maximizing the *log-likelihood function*

$$\mathcal{L} \rightarrow \log P(\mathbf{r}|\mathbf{v}) = \sum_i \log P(r_i|v_i). \quad (1.10)$$

4.15 Quadrature Amplitude Modulation System (QAM) or Quadrature Amplitude Shift Keying (QASK)

In MPSK, one symbol is distinguished from other by phase difference but all are of the same amplitude. So the end point of the signal vectors in signal space falls on the circumference of the circle. Here to distinguish one signal vector from another in the presence of noise will depend on the distance between the vector end points. It is hence rather apparent that we shall be able to improve the noise immunity of a system by allowing the signal vectors to differ, not only in their phase but also in amplitude. That is, it is an amplitude and phase shift keying system. Here direct modulation of carriers in quadrature is taking place (i.e. $\cos \omega_c t$ and $\sin \omega_c t$). Hence this system is known as QPSK or QASK or QAM.

As an example of M-QAM (M-ary QAM), let us consider 16 QAM, system. Here $M=16, N=4$. Transmit a symbol every T_b and 16 distinguishable symbols are needed. One possible geometrical representation (signal constellation) is shown in figure. In this configuration each signal point is equally distant from its nearest neighbors, this distance being $d = 2a$.

The points are placed symmetrically about the origin of the signal space to simplify the hardware design of the system while keeping the energy per signal near a minimum.

Let us assume that all 16 signals are equally likely. Because of the symmetry displayed in figure the average normalized energy of a signal is

$$E_s = \frac{1}{4} [(a^2 + a^2) + (9a^2 + a^2) + (a^2 + 9a^2) + (9a^2 + 9a^2)] = 10a^2$$

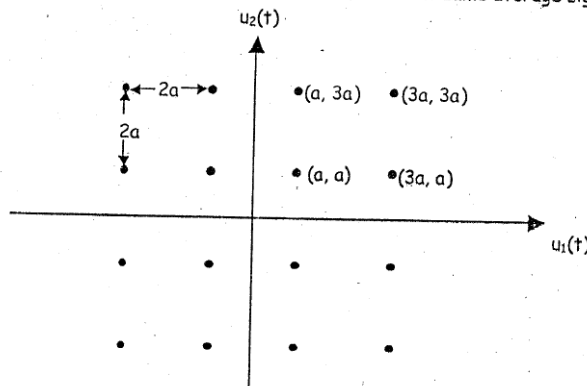
$$\therefore a = \sqrt{0.1E_s} \text{ and } d = 2a = 2\sqrt{0.1E_s}, \text{ Here } E_s = 4E_b \therefore a = \sqrt{0.4E_b}$$

$$d = 2a = 2\sqrt{0.4E_b}$$

$$\text{But for 16.PSK, } d = \sqrt{16E_b \sin^2\left(\frac{\pi}{16}\right)} = 2\sqrt{0.15E_b}$$

$$d_{16.PSK} < d_{16QAM} \text{ for the same } P_s \therefore P_e \text{ 16 QAM} < P_e \text{ 16 PSK for the same } P_s$$

Thus, 16 QAM will be shown to have a lower error rate than 16 PSK for the same average signal power.



Geometrical representation of 16 signals in a QASK

The set of message points corresponding to the set of transmitted signals ($s_1(t), s_2(t), \dots, s_{16}(t)$) is called a signal constellation

A typical QAM signal can be written as

$$V_{QAM}(t) = K_1 a u_1(t) + K_2 a u_2(t)$$

$$K_1, K_2 = \pm 1 \text{ or } \pm 3, a = \sqrt{0.1E_s}$$

$$u_1(t) = \sqrt{\frac{2}{T_s}} \cos \omega_c t, u_2(t) = \sqrt{\frac{2}{T_s}} \sin \omega_c t$$

$$\therefore V_{QAM}(t) = K_1 \sqrt{\frac{0.2E_s}{T_s}} \cos \omega_c t + K_2 \sqrt{\frac{0.2E_s}{T_s}} \sin \omega_c t$$

$$\text{But } \frac{E_s}{T_s} = P_s \therefore V_{QAM}(t) = K_1 \sqrt{0.2P_s} \cos \omega_c t + K_2 \sqrt{0.2P_s} \sin \omega_c t$$

$$= A_e(t) \sqrt{P_s} \cos \omega_c t + A_o(t) \sqrt{P_s} \sin \omega_c t$$

$$\text{Where } A_e, A_o = \pm \sqrt{0.2} \text{ or } \pm 3\sqrt{0.2}$$

