Discrete Time Signal Processing

QUE :1 a)

1. (a) A discrete time invariant and linear system is describe by the difference equation- 10

y(n) = x(n) + 2 x(n-1) + x{n- 2) Obtain
(i) Impulse response

(ii) Frequency response

(iii) Sketch magnitude and phase response

(iv) System response to the input {-1)^n n u(n).

Ans i) Impulse response

h(n) =[1 2 1]

or h(n)=δ(n)+ 2*δ(n-1)+ δ(n-2)

ii) Frequency response

$h(e^{jw})= 2e^{-jw}(1+\cos w)$

iii) Magnitude response & phase response

$| h(e^{jw}) |=2(1+\cos w)$

$< h(e^{jw})= -w$

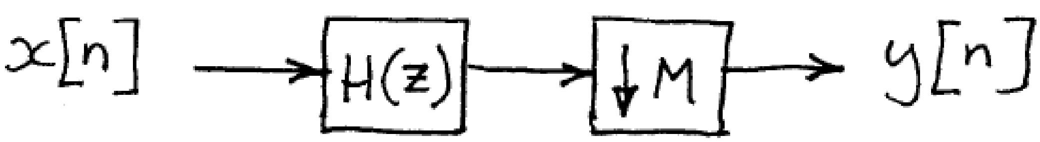
iv) System response to input $(-1)^n nu(n)$

y(n)=- δ(n+1) or

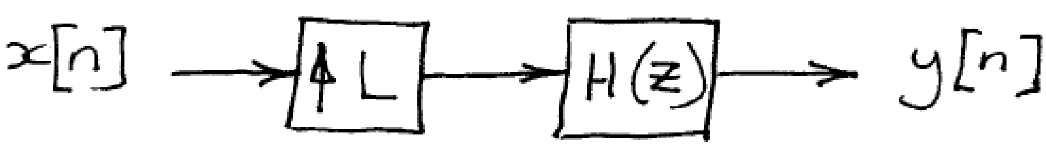
y(n)=[-1 0]

(b) Explain the concept of decimation by Integer (M) and interpolation by integer factor (L). 10

Fundamental Multirate Operations
Downsampling by a factor M :
filter and M-fold decimator

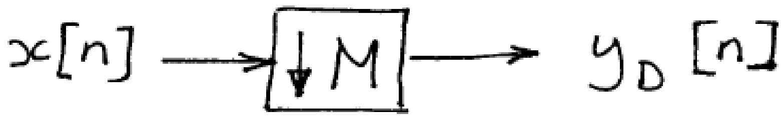


Upsampling by a factor L:
L-fold expander and filter

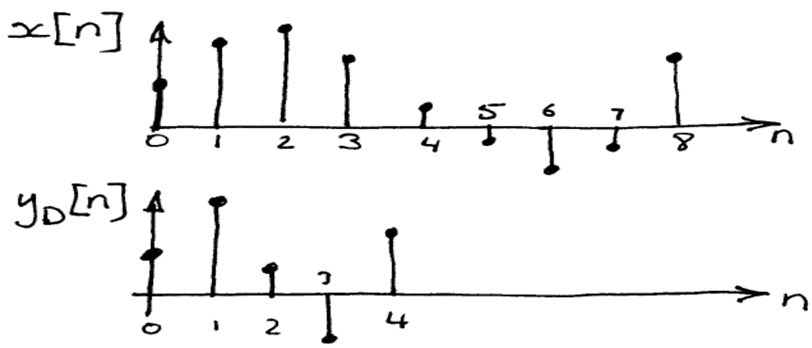


M-fold Decimator

For an input sequence x(n), select only the samples which occur at integer multiples of M. The other samples are thrown away



- yD(n) = x(Mn)
- Aliasing will occur in yD(n) unless x(n) is sufficiently bandlimited, loss of information.
  Eg. $M = 2$



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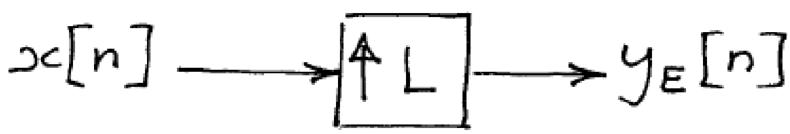
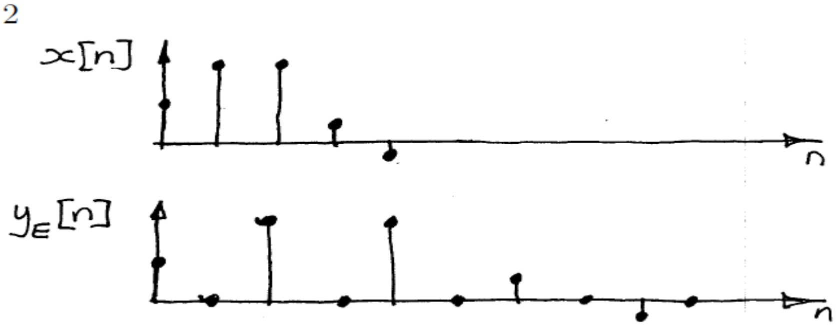
L-fold Expander
For an input sequence x(n), insert L – 1 zeros between each sample.



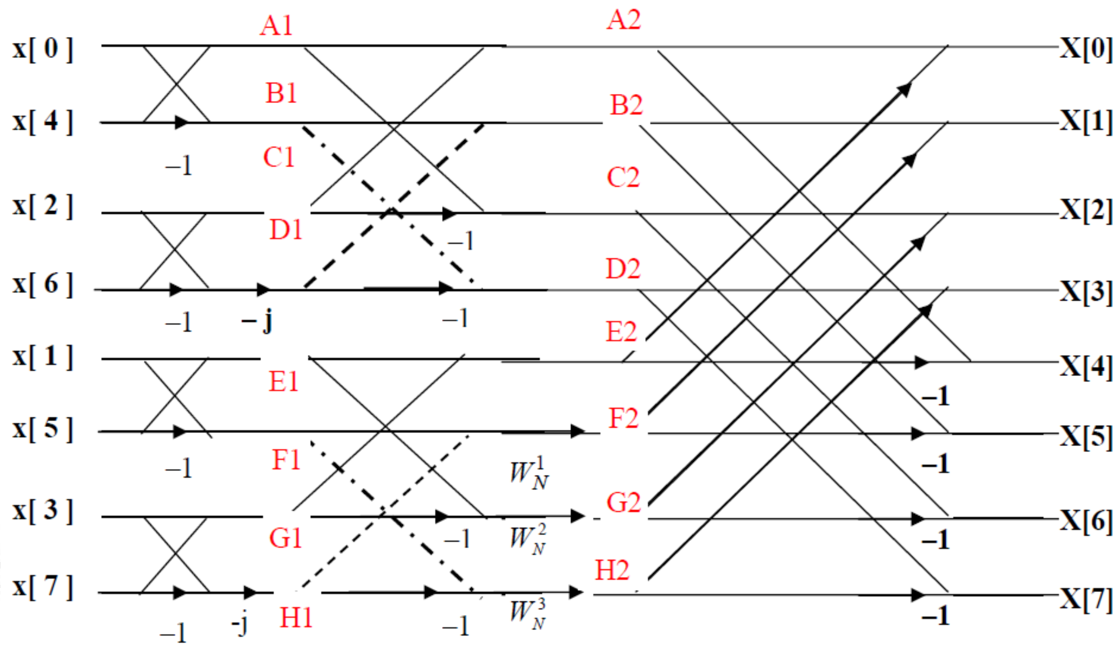
x(n) can always be recovered from yE(n)
no loss of information, no aliasing.

Eg. $L = 2$



Que 2a) Find DFT of the sequence using DIT FFT -

X(n)=[1 -2 2 2 1 3 -3 4]



X(k)=[8, -2.12-j.0051, 3+i5 2.12+j9.94, -6, 2.12-j9.94, 3-i5 , -2.12+j.0051]

Que 2 b) Convert the analog filter with system function-

$$h(s) = \frac{s+1}{9+(s+0.1)^2}$$

into a digital MR filter using bilinear transformation, the digital filter should have a resonant frequency of Wr=pi/4

Ans

Wr=pi/4 ,Ωc=3

Bilinear transformation

$\Omega c = \frac{2Tan\ (Wr/2)}{T}$

$3 = \frac{2Tan\ (pi/8)}{T}$

T=0.276

Digital filter can be found by replacing s by=$\frac{2(Z-1))}{T(Z+1)}$

Simplifying and putting T=0.276

H(Z)=$\frac{1+0.027z-1-0.973z-2}{8.5-11.84z-1+8177z-2}$

Que 3a) A filter is to be designed with the following desired frequency response

$$H_d(e^{jw}) = \begin{cases} 0 & ; \quad 0 \le w \le \frac{\pi}{4} \\ (e^{-j2w}) & ; \quad +\pi/4 < |w| < \pi \end{cases}$$

Determine the filter coefficients using Hamming window

Ans: delay τ= 2 ,delay τ= 2 = $\frac{M-1}{2}$

M=5

Applying inverse DFT

$H_d(n)$=1/2pi * $\{\int_{-pi}^{pi} h(e^{jwn})e^{jwn})\, dw \}$

=1/2pi * $\{\int_{-pi}^{pi/4}(e^{-j2w})e^{jwn})\, dw + \int_{pi/4}^{pi}(e^{-j2w})e^{jwn})\, dw \}$

== $\frac{\sin(n-2)pi-\sin(n-2)pi/4}{pi(n-2)}$

That implies

$H_d(n)$=[-0.15,-.22 ,0.75,-o.2251,-0.1592]

Hamming window =0.54-0.46cos(2*pi*n/M-1) ...for 0<= n <=M-1

=[0.08, 0.54,1 ,0.54, 0.08]

Filter coefficient

H(n)=hd(n)*Wh(n)

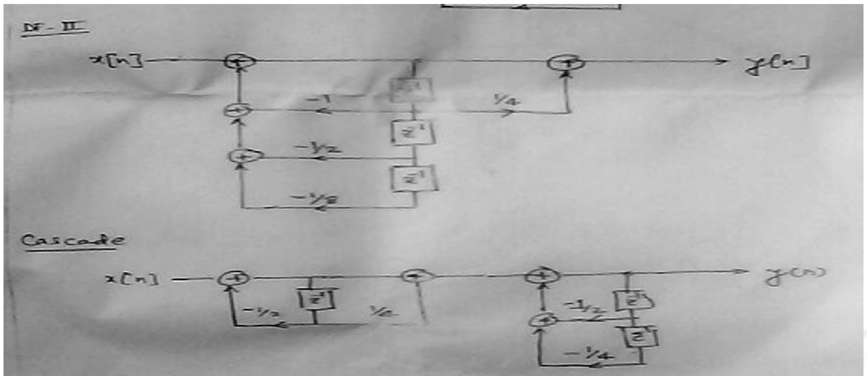
=[-0.012.-0.12,0.75,-0.12,-.0127]

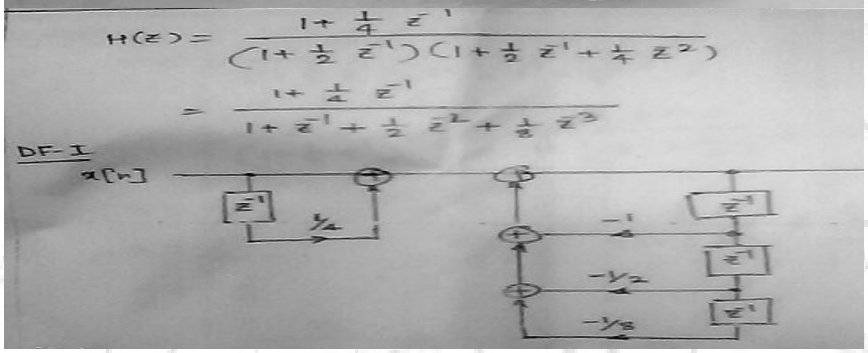
Q 3 b) Consider a causal LTI system which is defined by system function

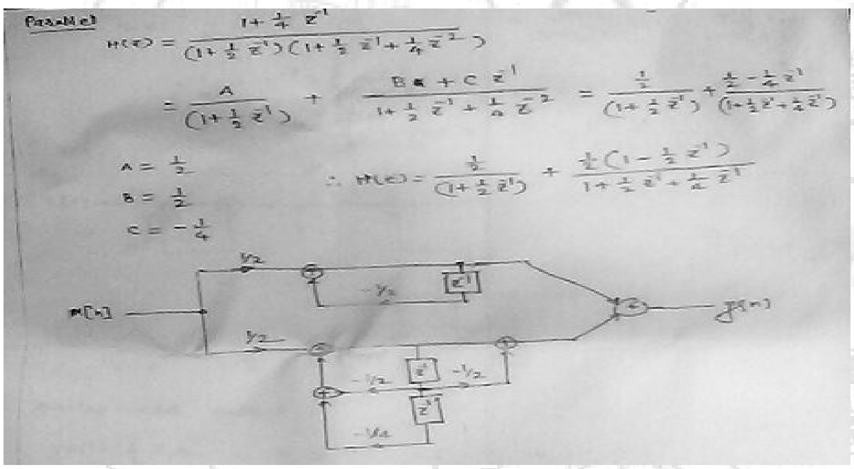
$$H(z) = \frac{1+\frac{1}{4}z^{-1}}{\left(1+\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}\right)}$$

Obtain DF-I, DF-II, cascade and parallel realization structures.

Discrete Time Signal Processing







Que 4 a) The frequency response of low pass filter is given by-

$$H(e^{jw}) = \begin{cases} (e^{-j3w}) \; ; & 0 \le w < \pi/2 \\ 0 \; ; & \pi/2 \le w \le \pi \end{cases}$$

Realize the above filter using frequency sampling realization technique

Ans : $H(e^{jw}) = e^{-j3w}$ for 0<w< pi/2

= 0 for pi/2 <w< pi

delay $\tau = 3 = \frac{M-1}{2}$

M=7

Frequency response samples at Wk=2*pi*k/M

Non zero sample exist for k= 0,1,6

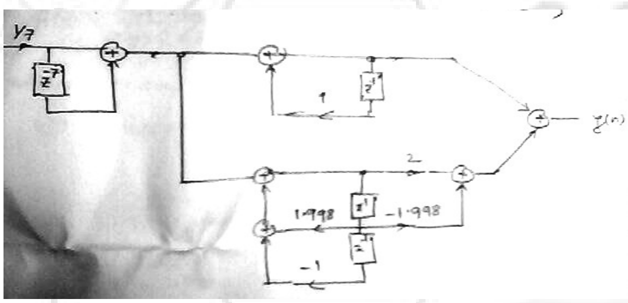
H(k)= $e^{-j6*pi*wk/7}$ for k=0,1

= $e^{-j6*pi*wk/7}$ for k=6

$$H(Z)=\frac{1}{M}\,[1-z^{-m}]*\left[\frac{h(o)}{1-w^{0}z^{-1}}+\frac{h(1)}{1-w^{-1}z^{-1}}+\frac{h(6)}{1-w^{-6}z^{-1}}\right]$$

$$=\frac{1}{7}\,[1-z^{-7}]*\left[\frac{h(o)}{1-z^{-1}}+\frac{h(1)}{1-e^{-j6*\frac{pi}{7}}z^{-1}}+\frac{h(6)}{1-e^{j6*\frac{pi}{7}}z^{-1}}\right]$$

$$=\frac{1}{7}\,[1-z^{-7}]*\left[\frac{1}{1-z^{-1}}+\frac{2-1.998z^{-1}}{1-1.998z^{-1}+z^{-2}}\right]$$



*Q4 (b) Develop DIT FFT algorithm for N = 6 = 2*3 using split-radix method.* ***10***

Solution (b)

By DFT, $X[k]=\sum_{n=0}^{N-1} x[n]\,W_N^{nk}$

By Decomposing 6 pt DFT into Three 2 pt DFT's

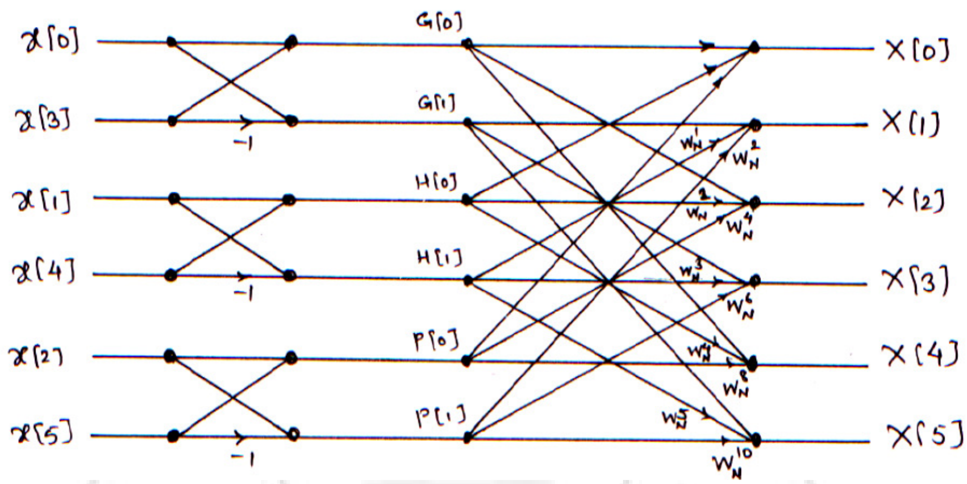
$$X[k]=\sum_{r=0}^{\frac{N}{3}-1} x[3r]\;W_N^{3rk}+\sum_{r=0}^{\frac{N}{3}-1} x[3r+1]\,W_N^{(3r+1)k}+\sum_{r=0}^{\frac{N}{3}-1} X[3r+2]\,W_N^{(3r+2)k}$$

$$X[k] = \sum_{r=0}^{\frac{N}{3}-1} x[3r]\, W_{\frac{N}{3}}^{rk} + W_N^k \sum_{r=0}^{\frac{N}{3}-1} x[3r+1]\, W_{\frac{N}{3}}^{rk} + W_N^{2k} \sum_{r=0}^{\frac{N}{3}-1} x[3r+2]\, W_{\frac{N}{3}}^{rk}$$

$$X[k] = G[k] + W_N^k\, H[k] + W_N^{2k}\, P[k].$$

Where $G[k] = DFT\{X[3r]\}$ $H[k] = DFT\{X[3r+1]\}$ $P[k] = DFT\{X[3r+2]\}$

$$G[k] = DFT\begin{bmatrix} X[0] \\ X[3] \end{bmatrix} \quad H[k] = DFT\begin{bmatrix} X[1] \\ X[4] \end{bmatrix} \quad P[k] = DFT\begin{bmatrix} X[2] \\ X[5] \end{bmatrix}$$



Q5 A)The unit sample response of a system is h[n) = { 3, 2, 1} use overlap-add method of linear filtering to determine output sequence for the repeating input sequence x [n] = { 2 , 0,-2,0,2, 1,0,-2,-1, 0} *10*

Ans Overlap and method

x1=[2, 0,-2,0]

x2=[2, 1,0,-2]

x3=[-1 0 0 0]

h=[3 2 1]

x1*h=[6 4 -4 -4 -2 0]

x2*h=[6 7 4 -5 -4 -2]

x3*h=[-3 -2 -1 0 0 0]

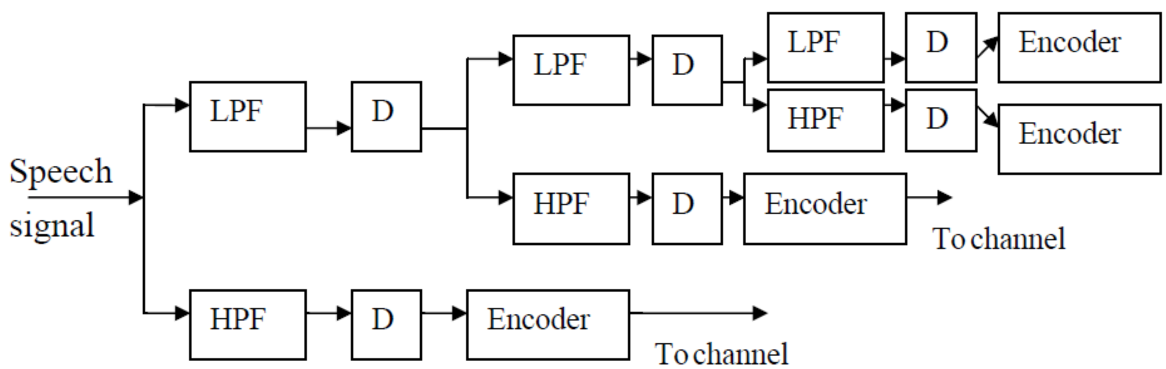
hence

Y(n)=[6 4 -4 4 4 7 4 -5 -7 - 4 -1 0]

(b) Explain the subband coding of speech signal as an application of mutirate signal processing *10*

Subband Coding

Consider quantizing the samples of a speech signal. How many bits are required?
In general, 16 bits precision per sample is normally used for audio. This gives an adequate dynamic range. In practice, certain frequency bands are less important perceptually because they contain less significant information bands with less information or lower perceptual importance may be quantized with lower precision - fewer bits. Divide the spectrum of the signal into several sub bands then allocate bits to each band appropriately.
16 bits per sample, 10 kHz sampling frequency gives160 kbits/s .Divide into 2 bands: high frequency and low frequency sub bands. High frequencies of speech are less important to intelligibility. Therefore use only 8 bits per sample The sampling frequency can be reduced by a factor of 2 since bandwidth is halved, still satisfying Nyquist criterion.5 × 16 + 5 × 8 = 120 kbits/s4:3 compression Reconstructed signal has no noticeable reduction is signal quality.



Que 6 a) Design a digital Butterworth filter that satisfies the following constraint using bilinear transformation Assume T = 1 sec. *10*

$$0.9 \le \left|H(e^{jw})\right| \le 1 \quad ; \quad 0 \le w \le \frac{\pi}{2}$$

$$\left|H(e^{jw})\right| \le 0.2 \quad ; \quad \frac{3\pi}{4} \le w \le \pi$$

Ans :

T=1

$\Omega = \frac{2Tan\ (W/2)}{T}$

W_1=pi/2 ,W_2=3pi/4

Ω1=2 rad/s

Ω2=4.828

Order of filter= 3

Ωc=2.54rad/s

Poles of analog filter

P1=1 $e^{2*pi/3}$

P1=1 e^{pi}

P3=1 $e^{4*pi/3}$

Corresponding transfer function $H_1(s)$=

$$\frac{1}{s^3+2s^{2+}2s+1}$$

Un Normalised analog filter

$H(s)=H_1(s)|s=s/\Omega c$

$=16.51/S^3+5.09S^2+12.97S+16.51$

Digital filter

$H(z)=H(s)|s=\frac{2(Z-1))}{T(Z+1)}$

$=0.23(1+z^{-1})^3/1+0.43z^{-1}+0.38z^{-2}+0.0416z^{-3}$

Que 6 b) Draw pole-zero plot and sketch magnitude and phase response of finite impulse response fitter which is given by. 10

$$h[n] = (0.5)^n \quad ; \quad 0 <= n <= 7$$

Ans

$h(n)=(0.5)^n$ for 0 <= n <=7

$H(z)=\sum_{n=-inf}^{inf} h(n)z^{-n}$

$=\sum_{n=0}^{7}(0.5)^n z^{-n}$

$=\sum_{n=0}^{7}(0.5z^{-1})^n$

$=\frac{1-0.5^8 z^{-8}}{1-0.5z^{-1}}$

$=\frac{z^8-0.5^8}{z^7(z-0.5)}$

Equating numerator to zero

$z^8 = 0.5^8.1=0.5^8 e^{i2*pi*k}$

$z^8 = 0.5^1 e^{i2*pi*k/8}$

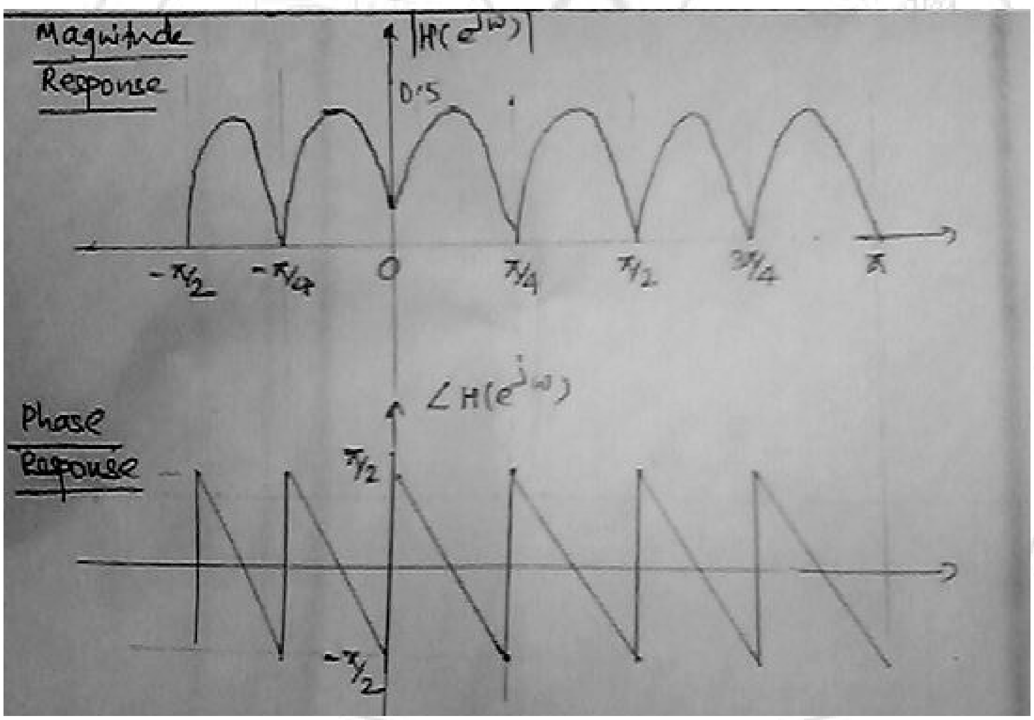
That implies

$Z_{0=}$ $0.5e^{j0}$ $Z_{1=}$ $0.5e^{j*pi/4}$ $Z_{2=}$ $0.5e^{pi*j*/2}$ $Z_{3=}$ $0.5e^{3j*pi/4}$ $Z_{4=}$ $0.5e^{pi*j}$

$Z_{5=}$ $0.5e^{5*j*pi/4}$ $Z_{6=}$ $0.5e^{3j*pi/2}$ $Z_{7=}$ $0.5e^{7j*pi/4}$

Seven poles at origin one pole at 0.5

One zero at 0.5 and one pole at 0.5 cancel each other



7. *Write short note on (any* ***two****) :—* 20

(a) *Multistage approach to sampling rate conversion.*
(b) *Adaptive television echo cancellation.*

(c) Goertzel Algorithm
(d) Digital resonator.

c) The Goertzel Algorithm

The Goertzel Algorithm makes use of the periodicity of the sequence Wjf to reduce the computation involved in calculating DFT.

By DFT, $X[k]=\sum_{n=0}^{N-1}x[n]\,W_N^{nk}=\sum_{r=0}^{N-1}x[r]\,W_N^{rk}$

Multiplying by W_N^{-kN}

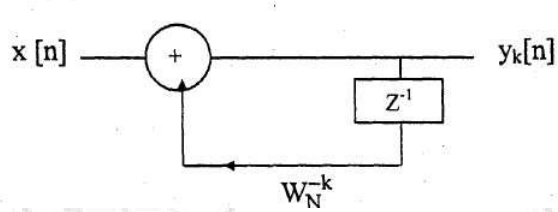
$$X[k]=W_N^{-kN}\sum_{r=0}^{N-1}x[r]\;W_N^{rk}=\sum_{r=o}^{N-1}x[r]\;W_N^{rk}\;W_N^{-kN}$$

$$=\sum_{r=0}^{N-1}x[r]\;W_N^{-k(N-r)}$$

Let $y_k[n]=\sum^{N-1}x[r]\,W_N^{-k(n-r)}=x[n]*W_N^{-kn}\;u[n].$

yk[n] can be veined as the response of the system with impulse response, h_k [n]* W_N^n u[n].

By ZT, $H_k(z)=\dfrac{Z}{Z-W_N^{-k}}$

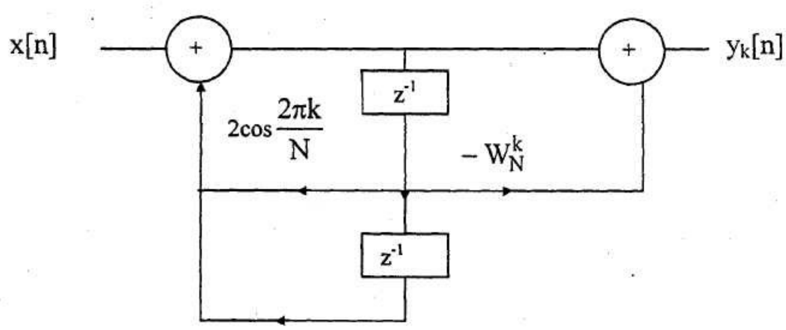


(i) To find difference equation,

$$H_k(z)=\frac{z}{(z-w_N^{-k})}\,\frac{z-w_N^k}{z-w_N^k}=\frac{z^2-w_N^k\,z}{z^2-2\cos\left(\frac{2\pi k}{N}\right)z+1}$$

d) Digital resonator.

(ii) Realization diagram of second order system is given by,



A digital resonator is a special two pole and-pass filter with the pair of complex conjugate poles located near the unit circle as shown below. Digital resonator is a narrowband band-pass filter. The name resonator refers to the fact that the filter has a large magnitude response (ie it resonates) in the vicinity of the pole location. The angular position determines the resonant frequency of the filter. Digital resonators are useful for bandpass filtering, speech generation etc.

Design of digital resonator : To have a peak at $at = w_0$ select the complex conjugate poles at p1 = r exp(jwo) and p2 = r exp(-jwo) and zeros at Z1 = 1 and Z_2 = -1

$$H(z) = \frac{G(z-1)(z+1)}{(z-re^{j\omega_o})(z-re^{-j\omega_o})} = \frac{G(z-1)(z+1)}{z^2 - 2\,r\,z\cos(\omega_o) + r^2}$$