Q. 1) a) *Obtain half range sine series to represent:* 05

$$f(x)=\frac{2x}{3},0\le x\le\frac{\pi}{3}$$
$$=\frac{\pi-x}{3},\frac{\pi}{3}\le x\le\pi$$

Ans:

$$bn=\frac{2}{\pi}\int_0^{\pi} f(x)\sin n\pi x dx$$ 01

$$=\frac{2}{\pi}\left[\int_0^{\pi/3}\frac{2x}{3}\sin n\pi x dx+\int_{\pi/3}^{\pi}\frac{\pi-x}{3}\sin n\pi x dx\right]$$ 02

03

$$=\frac{2}{\pi}\frac{1}{n^2}\sin\frac{n\pi}{3}$$

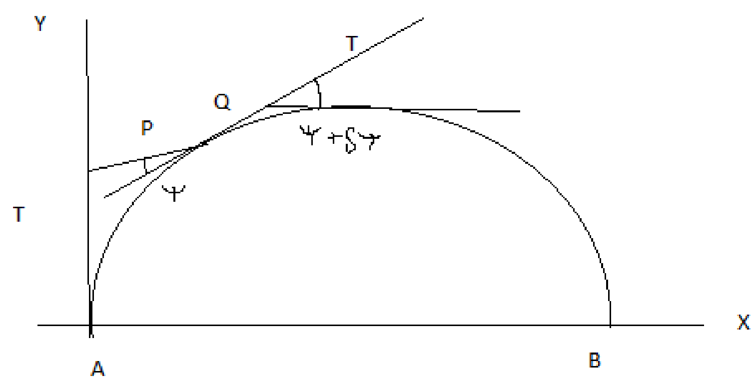
$$so, f(x)=\frac{2}{\pi}\sum\frac{1}{n^2}\sin\frac{n\pi}{3}\sin nx$$ 05

b) *Derive one dimensional wave equation stating all conditions.*

Ans: Let at time t the position of the string be shown in fig. consider the motion of the element PQ of the string between two points P(x,y)and Q(x+□x, y+□y). Let the tangents at P & Q makes angles □+□□ with x-axis resp.

01

The element PQ is *moving* vertically upwards with acceleration $\frac{\partial^2 y}{\partial t^2}$.



02

Since the string does not offer resistance to bending, vertical component of the tension T on the elt. Is:

$$= T\sin(\varphi+\delta\varphi) - T\sin\varphi$$ 04
$$= T[\sin(\varphi+\delta\varphi) - \sin\varphi]$$
$$= T[\tan(\varphi+\delta\varphi) - T\tan\varphi]$$
$$= T\left[\frac{\partial y}{\partial x} at(x+\delta x) - \frac{\partial y}{\partial x} at(x)\right]$$

Let m be mass per unit length of the string. Then by Newton's 2^{nd} law of motion, we write:

$$m.\delta x.\frac{\partial^2 y}{\partial t^2} = T\left[\frac{\partial y}{\partial x} at(x+\delta x) - \frac{\partial y}{\partial x} at(x)\right]$$
$$\frac{\partial^2 y}{\partial t^2} = \frac{T}{m}\left[\frac{\frac{\partial y}{\partial x} at(x+\delta x) - \frac{\partial y}{\partial x} at(x)}{\delta x}\right]$$

$$At Q \longrightarrow P,$$
$$\delta x \longrightarrow 0$$ 05
$$and$$
$$\frac{T}{m} = c^2,$$
$$\frac{\partial^2 \mathbf{y}}{\partial \mathbf{t}^2} = \mathbf{c}^2 \frac{\partial^2 \mathbf{y}}{\partial \mathbf{x}^2}$$

c) *If X denotes the outcomes when a fair die is tossed, find m,g,f, of X and hence find mean and variance of X.*

Ans: p.d.f.:

X	1	2	3	4	5	6
P(x)	1/6	1/6	1/6	1/6	1/6	1/6

01

$$m.g.f. = \sum pe^{tx}$$

$$= \frac{1}{6}\left[e^{t} + e^{2t} + e^{3t} + e^{4t} + e^{5t} + e^{6t}\right]$$

$$\frac{dm}{dt} = \frac{1}{6}\left[e^{t} + 2e^{2t} + 3e^{3t} + 4e^{4t} + 5e^{5t} + 6e^{6t}\right]$$ 02

$$\frac{dm}{dt}(at = 0) = \frac{1}{6}[1+2+3+4+5+6]$$

$$= \frac{1}{6}\frac{6.7}{2} = \frac{7}{2}.$$

$$so, M = \frac{7}{2}$$ 03

$$\frac{d^2m}{dt^2} = \frac{1}{6}\left[e^{t} + 4e^{2t} + 9e^{3t} + 16e^{4t} + 25e^{5t} + 36e^{6t}\right]$$

$$\frac{d^2m}{dt^2}(at = 0) = \frac{1}{6}[1+4+9+16+25+36] = \frac{91}{6}$$ 04

$Now,$

$$V(x) = E(X^2) - [E(X)]^2$$

$$= \frac{91}{6} - \left(\frac{7}{2}\right)^2 = 2.91$$ 05

d) *Find the regression coefficients and the coefficient of correlation from the following data:*

$$N = 12, \sum x = 120, \sum y = 432, \sum xy = 4992, \sum x^2 = 1392, \sum y^2 = 18252$$

Ans:

$$b_{yx} = \frac{\sum xy - \frac{\sum x.\sum y}{N}}{\sum x^2 - \frac{\left(\sum x\right)^2}{N}}$$

$$= \frac{4992 - \frac{120\times 432}{12}}{1392 - \frac{(120)^2}{12}} = 3.5$$ 02

$$b_{yx} = \frac{\sum xy - \frac{\sum x.\sum y}{N}}{\sum y^2 - \frac{\left(\sum y\right)^2}{N}}$$

$$= \frac{4992 - \frac{120\times 432}{12}}{18252 - \frac{(432)^2}{12}} = 0.25$$ 04

$so,$

$$r = \sqrt{b_{yx} \times b_{yx}}$$

$$= \sqrt{3.5\times 0.25}$$ 05

$$= 0.94$$

Q. 2) a) *Expand f(x)=|x| in (-π,π) into a fourier series.*

Ans: $f(x) = -x, -\pi < x < 0$ 01

$= x, 0 < x < \pi$

$$a0 = \frac{1}{2\pi}\left[\int_{-\pi}^{0} -x dx + \int_{0}^{\pi} x dx\right]$$ 02

$$= \frac{\pi}{2}.$$

$$an = \frac{1}{\pi}\left[\int_{-\pi}^{0} -x\cos nx dx + \int_{0}^{\pi} x\cos nx dx\right]$$

$$= \frac{2}{\pi n^2}(\cos n\pi - 1)$$ 05

$$f(x) = \frac{\pi}{2} - \frac{4}{2}\left[\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + ...\right]$$ 06

b) *The following data gave the growth of employment in lakhs in organized sector in India between 1988 and 1995.* 06

Year	1988	89	90	91	92	93	94	95
Public sector	98	101	104	107	113	120	125	128
Private sector	65	65	67	68	68	69	68	68

Find the correlation coefficient between the employment in public and private sector.

Ans:

$\bar{x}$ =112			$\bar{y}$ =67			$(x-\bar{x})$.
x	$x-\bar{x}$	$(x-\bar{x})^2$	y	$y-\bar{y}$	$(y-\bar{y})^2$	$(y-\bar{y})$
98	-14	196	65	-2	4	28
101	-11	121	65	-2	4	22
104	-8	64	67	0	0	0
107	-5	25	68	1	1	-5
113	1	1	68	1	1	1
120	8	64	69	2	4	16
125	13	169	68	1	1	13
128	16	256	68	1	1	16
		896			**16**	**91**

03

So,

$$r=\frac{\sum\left(x-\bar{x}\right)\left(y-\bar{y}\right)}{\sqrt{\sum\left(xi-\bar{x}\right)^2\sum\left(yi-\bar{y}\right)^2}}$$ 04

$$=\frac{91}{\sqrt{896\times16}}=0.77$$ 06

c) *A homogeneous rod of conducting material of length l has ends kept at zero temperature and the temperature at the centre is T and falls uniformly to zero at the two ends. Find the temperature u(x,t)at any time.* 08

Ans: The equation of heat flow is:

$u_t = c2\ u_{xx}$ ....(1)

so, temp. at two ends are zero its solution is:

$u=(c1 \cos mx + c2 \sin mx)\ e^{-m2c2t}$ ....(2) 02

ics.:

(i) U=0, x=0,

$C1\ e^{-m2c2t} = 0$. So, c1 = 0. 03

Equ. (ii) becomes,

$u=(c2 \sin mx)\ e^{-m2c2t}$ ....(iii)

(ii) U=0 when x=l

$0=c2 \sin mx)\ e^{-m2c2t}$

$ml=n\pi$ 05

so, $m = n\pi/l$, n=1,2,3,......

so, general solution is:

$$u = \sum bn \sin \frac{n\pi x}{l} e^{-n^2c^2\pi^2 t/l^2}$$

(iii) When t=0 temp at centre is I and falls uniformly to zero at the two ends.

$$u = \frac{2T}{l} x, 0 < x \le l/2$$
$$= \frac{2T}{l}(l - x), l/2 \le x < l$$

Hence putting t=0 we get,

$$f(x) = \sum bn \sin \frac{n\pi x}{l}$$ 06

But this is half range sine series for f(x) in range (0,l)

Where,

$$bn = \frac{2}{l}\int_0^{\pi} f(x)\sin\frac{n\pi x}{l}dx$$ 07

$$= \frac{2}{l}\left[\int_0^{l/2}\frac{2T}{l}x\sin\frac{n\pi x}{l}dx + \int_{l/2}^{l}\frac{2T}{l}(l-x)\sin\frac{n\pi x}{l}dx\right]$$

$$= \frac{8T}{\pi^2}\frac{1}{n^2}\sin\frac{n\pi}{2}$$

$$so, u = \frac{8T}{\pi^2}\sum\frac{1}{n^2}\sin\frac{n\pi}{2}\sin\frac{n\pi x}{l}e - n^2c^2\pi^2t/l^2$$ 08

Q.3) a) *The number of accidents in a year attributed to taxi drivers in a city follws Poisson distribution with mean 3. Out of 1000 taxi drivers, find approximately the number of drivers with (i) no accidents in a year, (ii) more than 3 accidents in a year. (given: e^{-1} = 0.3679, e^{-2}=0.1353, e^{-3} = 0.0498).* 07

Ans:

$$P(x) = \frac{e^{-m}m^x}{x!}, x = 0,1,2,3......$$

$$given, m = 3$$

$$P(x) = \frac{e^{-3}3^x}{x!}$$ 01

P(x=0)= e^{-3} =0.498

P(x=1)=0.1494

P(x=2)=0.2241 03

So, expected no. of drivers with no accidents

= N. P(0)

= 1000. 0.0498 = 49.8 = 50 nearly 05

P(0,1,2,3 accidents)=p(0) + p(1) + p(2) + p(3) = 0.4233

Therefore, p(more than 3 accidents) = 1-0.4233 = 0.5767 06

Expected no. of drivers with more than 3 accidents= NP =1000. 0.5767

=577 nearly 07

b) *Obtain the Fourier series for f(x) = sin2x in (-l, l)* 07

Ans: Here, f(-x) = sin(-2x)= - sin2x = -f(x)

So, function is odd function.

i.e. a0=an=0 02

now,

$$bn = \frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}dx$$

$$= \frac{2}{l}\int_0^l \sin 2x \sin\frac{n\pi x}{l}dx$$ 03

$$= \frac{-1}{l}\left[\int_0^l \cos\left(2+\frac{n\pi}{l}\right)x - \cos\left(2-\frac{n\pi}{l}\right)xdx\right]$$

$$= \frac{-1}{l}\left[\frac{1}{2l+n\pi}\sin\left(2+\frac{n\pi}{l}\right)x - \frac{1}{2l-n\pi}\sin\left(2-\frac{n\pi}{l}\right)x\right]_0^l$$

$$= -\left[\frac{1}{2l+n\pi}\sin(2l+n\pi)x - \frac{1}{2l-n\pi}\sin(2l-n\pi)x\right]$$

$$= -\left[\frac{\sin al\cos n\pi}{al+n\pi} - \frac{\sin al\cos n\pi}{al-n\pi}\right]$$

$$= \frac{-2n\pi}{a^2l^2-n^2\pi^2}\sin al\cos n\pi$$ 05

$$= \frac{-2n\pi(-1)^n}{a^2l^2-n^2\pi^2}\sin al$$

$$so, f(x) = 2\pi\sin al\sum\frac{-2n\pi(-1)^n}{a^2l^2-n^2\pi^2}\sin\frac{n\pi x}{l}$$ 07

c) *Fit the second degree curve to the following data and estimate the value of y when x = 80.* 06

X	10	20	30	40	50	60	70
y	20	60	70	80	90	100	100

Ans:

x	(x-40)/10	X^2	X^3	X^4	y	y/10	xy	X^2y
10	-3	9	27	81	20	2	-6	18
20	-2	4	8	16	60	6	-12	24
30	-1	1	1	1	70	7	-7	7
40	0	0	0	0	80	8	0	0
50	1	1	1	1	90	9	9	9
60	2	4	8	16	100	10	20	40
70	3	9	27	81	100	10	30	90

03

	0	**28**	**72**	**196**		**52**	**34**	**188**

$Y = 6.095 + 0.9286x + 0.0476\, x^2$ 05

If x = 80, **y = 105.71** 07

Q.No.4.a. Determine in two different ways, the probability that by guesswork a student can correctly answer 25 to 30 questions in a multiple choice quiz consisting of 80 questions. Assume that in each question with four choices only one is correct and the student has no knowledge. *6*

Ans: (i) by Normal distribution:

$$m = np = 80 \times \frac{1}{4} = 20, q = \frac{3}{4}$$

$$\sigma = \sqrt{npq} = 3.873 \qquad (1)$$

$$z = \frac{X - m}{\sigma} = \frac{X - 20}{3.873}$$

$$X : 25 \to 30$$

$$: 24.5 \to 30.5$$

$$X = 24.5 \to z = 1.16 \qquad (2)$$

$$X = 30.5 \to z = 2.71$$

$$P(24.5 \le x \le 30.5) = P(1.16 \le z \le 2.71)$$

$$= 0.4966 - 0.6770 \qquad (3)$$

$$= 0.1196$$

(ii) By Binomial Distribution

$$p = \frac{1}{4}, q = \frac{3}{4}, n = 80$$

$$P(X = x) = {}^{n}c_x\, p^x q^{80-x}, x = 0, 1, 2, 3, \ldots\ldots, n \qquad (4)$$

$$= {}^{80}c_x \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{80-x}$$

$$= \sum_{X=25}^{30} {}^{80}c_x \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{80-x}$$ *(5)*

$$= 0.0434 + 0.0306 + 0.0204 + 0.0129 + 0.0077 + 0.0043$$

$$= 0.1193$$ *(6)*

Q.No.4.b. A tightly stretched string with fixed end points x = 0 and x = l is initially in equilibrium position. It is set vibrating by giving to each of its points velocity. *6*

$\left(\frac{\partial y}{\partial t}\right)_{t=0} = v_0 \sin^3\left(\frac{\pi x}{l}\right)$. *Find y(x ,t) .*

Ans: The vibration of string (wave equation) is given by : $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$

Its general solution is

$$y = (c_1 \cos mx + c_2 \sin mx)(c_3 \cos mct + c_4 \sin mct)$$ -------------(1) *(1)*

(i) when x = 0 y = 0 for all t (at A)

$$0 = (c_1)(c_3 \cos mct + c_4 \sin mct)$$

$\therefore c_1 = 0$ putting in eq.(1)

$$y = (c_2 \sin mx)(c_3 \cos mct + c_4 \sin mct)$$ -------------(2)

(ii) when t= 0 ,y = 0 foe all x (along x-axis)

$$0 = (c_2 \sin mx)(c_3)$$

$\therefore c_3 = 0$ putting in eq (2) *(2)*

$$y = (c_2 \sin mx)(c_4 \sin mct)$$

$$y = c_2 c_4 \sin mx \sin mct$$

$$y = c_5 \sin mx \sin mct$$ -------------(3)

where $c_2 c_4 = c_5$

(iii) when x = l , y = 0 for all t (at B)

$$0 = c_5 \sin ml \sin mct$$

$$\therefore c_5 = 0 \text{ or } \sin ml = 0$$

But $c_5 = 0$ gives trivial solution (From eq 3.)

$$\therefore \sin ml = 0$$

$$\therefore ml = n\pi \quad \text{where } n = 1, 2, 3, ...$$ *(3)*

$$\therefore m = \frac{n\pi}{l}$$ putting in eq. (3)

$$y = c_5 \sin \frac{n\pi x}{l} \sin \frac{n\pi ct}{l} \qquad \text{where } n = 1, 2, 3, ...$$

$$y = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l} \sin \frac{n\pi ct}{l}$$ ---------(4)

(iv) When t=0, $\left(\frac{\partial y}{\partial t}\right) = v_0 \sin^3\left(\frac{\pi x}{l}\right)$

$$\frac{\partial y}{\partial t} = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l} \cos \frac{n\pi ct}{l} \left(\frac{n\pi c}{l}\right)$$

$$\frac{\partial y}{\partial t} = \sum_{n=1}^{\infty} b_n \left(\frac{n\pi c}{l}\right) \sin \frac{n\pi x}{l} \cos \frac{n\pi ct}{l}$$

Putting t=0, $\left(\frac{\partial y}{\partial t}\right) = v_0 \sin^3\left(\frac{\pi x}{l}\right)$

$$v_0 \sin^3\left(\frac{\pi x}{l}\right) = \sum_{n=1}^{\infty} b_n \left(\frac{n\pi c}{l}\right) \sin \frac{n\pi x}{l}$$ *(4)*

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$

$$\sin^3\theta = \frac{3}{4}\sin\theta - \frac{1}{4}\sin 3\theta$$

$$v_0\left[\frac{3}{4}\sin\left(\frac{\pi x}{l}\right) - \frac{1}{4}\sin\left(\frac{3\pi x}{l}\right)\right] = \sum_{n=1}^{\infty} b_n \left(\frac{n\pi c}{l}\right) \sin \frac{n\pi x}{l}$$

$$v_0\left[\frac{3}{4}\sin\left(\frac{\pi x}{l}\right)-\frac{1}{4}\sin\left(\frac{3\pi x}{l}\right)\right]=b_1\left(\frac{\pi c}{l}\right)\sin\frac{\pi x}{l}+$$

$$b_2\left(\frac{2\pi c}{l}\right)\sin\frac{2\pi x}{l}++b_3\left(\frac{3\pi c}{l}\right)\sin\frac{3\pi x}{l}+\ b_4\left(\frac{4\pi c}{l}\right)\sin\frac{4\pi x}{l}+---$$

Comparing,

$$b_1\frac{\pi c}{l}=v_0\frac{3}{4},b_2=0,b_3\frac{3\pi c}{l}=v_0\frac{-1}{4},b_4=0,b_5=0,--- \quad (5)$$

$$b_1=\frac{3v_0 l}{4\pi c},b_3=\frac{-v_0 l}{12\pi c},b_2=b_4=b_5=---=0$$

Putting in eq.(4)

$$y(x,t)=\frac{3v_0 l}{4\pi c}\sin\frac{\pi x}{l}\sin\frac{\pi ct}{l}+0-\frac{v_0 l}{12\pi c}\sin\frac{3\pi x}{l}\sin\frac{3\pi ct}{l}+0+0+0+---$$

$$y(x,t)=\frac{3v_0 l}{4\pi c}\sin\frac{\pi x}{l}\sin\frac{\pi ct}{l}-\frac{v_0 l}{12\pi c}\sin\frac{3\pi x}{l}\sin\frac{3\pi ct}{l} \quad (6)$$

Q.No.4.c. Find Fourier series for f(x) in (0,2π)

$$f(x)=x, \qquad 0<x<\pi$$
$$=2\pi-x, \qquad \pi\le x\le 2\pi$$

Hence, deduce that $\frac{\pi^4}{96}=\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\frac{1}{7^4}+---$ *8*

Ans:

$$f(x)=a_0+\sum_{n=1}^{\infty}a_n\cos nx+\sum_{n=1}^{\infty}b_n\sin nx\ \forall x\in[0,2\pi] \quad (1)$$

$$a_o=\frac{1}{2\pi}\int_0^{2\pi}f(x)dx=\frac{1}{2\pi}\left[\int_0^{\pi}xdx+\int_{\pi}^{2\pi}(2\pi-x)\,dx\right]=\frac{\pi}{2} \quad (3)$$

$$a_n=\frac{1}{\pi}\int_0^{2\pi}f(x)\cos nxdx \quad (4)$$

$$a_n = \frac{1}{\pi}\left[\int_0^{\pi} x\cos nx dx + \int_{\pi}^{2\pi}(2\pi - x)\cos nx dx\right]$$

$$a_n = \frac{-2\left[1-(-1)^n\right]}{\pi n^2} \qquad (5)$$

$$b_n = \frac{1}{\pi}\left[\int_0^{\pi} x\sin nx dx + \int_{\pi}^{2\pi}(2\pi - x)\sin nx dx\right]$$

$$b_n = 0\,. \qquad (6)$$

Deduction:

$$\frac{1}{2\pi}\int_0^{2\pi}[f(x)]^2\,dx = {a_0}^2 + \frac{1}{2}\sum_{n=1}^{\infty}\left({a_n}^2 + {b_n}^2\right)$$

$$\frac{1}{2\pi}\int_0^{2\pi}[f(x)]^2\,dx = \frac{1}{2\pi}\int_0^{\pi}[x]^2\,dx + \frac{1}{2\pi}\int_{\pi}^{2\pi}[2\pi - x]^2\,dx \qquad (7)$$

$$\frac{1}{2\pi}\int_0^{2\pi}[f(x)]^2\,dx = \frac{\pi^3}{3}$$

$$\frac{\pi^3}{3} = \left(\frac{\pi}{2}\right)^2 + \frac{1}{2}\sum_{n=1}^{\infty}\left[\frac{-2\left[1-(-1)^n\right]}{\pi n^2}\right]^2$$

$$\frac{\pi^3}{3} = \left(\frac{\pi^2}{4}\right) + \frac{4}{2}\sum_{n=1}^{\infty}\left[\frac{\left[1-(-1)^n\right]^2}{\pi^2 n^4}\right]$$

$$\therefore \frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + --- \qquad (8)$$

Q.No.5. a. Solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial y} + 2u$ *by the method of separation of variables, given u = 0 and* $\frac{\partial u}{\partial x} = 1 + e^{-3y}$ *when x = 0.*

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Ans:

$$\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial y}+2u \quad \text{-----------------(1)}$$

Let $u = X \cdot Y$ be the solution of (1), Where X is function of x only and Y is function of y only.

$$\frac{\partial u}{\partial x}=X'\cdot Y \qquad \frac{\partial u}{\partial y}=X\cdot Y' \qquad (1)$$

$$\frac{\partial^2 u}{\partial x^2}=X''\cdot Y$$

Putting the values in eq(1),

$$X''\cdot T = X\cdot Y' + 2XY$$

$$\frac{X''-2X}{X}=\frac{Y'}{Y} \quad \text{----------------(2)} \qquad (2)$$

Clearly LHS of(2) is function of x only and RHS of (2) is function of y only.

Since x and y are independent variables , the eq(2) hold good if each side is equal to constant k (say).

$$\frac{X''-2X}{X}=\frac{Y'}{Y}=a$$

$$\frac{X''-2X}{X}=a \qquad \frac{Y'}{Y}=a$$

$$X''-(2+a)X=0 \qquad Y'-aY=0 \qquad (3)$$

$$\frac{d^2X}{dx^2}-(2+a)X=0 \qquad \frac{dY}{dy}-aY=0$$

Putting $d/dx = D$ putting $d/dy = D$

$$D^2X-(2+a)X=0 \qquad DY-aY=0$$

$$\left(D^2-(2+a)\right)X=0 \qquad (D-a)Y=0 \qquad (4)$$

Auxillary equation :

$$\left(D^2-(2+a)\right)=0 \qquad \left(D-a\right)=0$$

$$D^2=2+a \qquad D=a$$

$$D=\pm\sqrt{a+2}$$

$$X=c_1e^{\left(\sqrt{a+x}\right)x}+c_2e^{-\left(\sqrt{a+x}\right)x} \qquad Y=c_3e^{ay}$$

$$u=X\cdot Y=\left[c_1e^{\left(\sqrt{a+x}\right)x}+c_2e^{-\left(\sqrt{a+x}\right)x}\right]\cdot c_3e^{ay} \qquad (5)$$

Using boundary conditions
u = 0 and
$\frac{\partial u}{\partial x}=1+e^{-3y}$ when x = 0.

On simplification

$$u=\frac{1}{\sqrt{2}}\cdot\sinh\left(\sqrt{2}\cdot x\right)+\sin x\cdot e^{-3y} \qquad (6)$$

Q.No.5.b.A random sample of size 36 has 53 as mean and sum of squares of deviations from mean is 150.Can this sample be regarded as drawn from the population having 54 as mean? 6

Ans: $n=36,\bar{x}=53,\mu=54$ (1)

$$\sqrt{\sum\left(x_i-\bar{x}\right)^2}=\sqrt{150}$$

$$\sigma=\sqrt{\frac{\sum\left(x_i-\bar{x}\right)^2}{n}}=2.0412 \qquad (2)$$

Null Hypothesis: $\mu=54$

Alternative hypothesis: $\mu\neq54$ (3)

Test Statistics: $Z=\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}=-2.94$, $|Z|=2.94$ (4)

LOS: $\alpha=5\%$

Critical value: At $\alpha=5\%\rightarrow Z_\alpha=1.96$ $|z|=2.94>Z_\alpha$ (6)

Decision: The null hypothesis is rejected.

Q.No. 5.c.The regression equation of y on x and x on y are respectively y = x and 4x-y=3 and the second moment of x about the origin is 2.Find the mean of x and y.(ii)correlation coefficient. (iii)standard deviation of x and y.

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Ans: Regression equation of y on x: $Y = X$ *(1)*

$$\therefore b_{yx} = r\frac{\sigma_y}{\sigma_x} = 1$$

Regression equation of x on y: $X = \frac{1}{4}Y + 3$

$$b_{xy} = r\frac{\sigma_x}{\sigma_y} = \frac{1}{4} \qquad (2)$$

$$Y = X \quad , \quad X = \frac{1}{4}Y + 3 \qquad \Rightarrow X = 1, Y = 1 \qquad (3)$$

$$i.e.\overline{X} = 1, \overline{Y} = 1$$

$$r = \left(b_{yx} \times b_{xy}\right)^{1/2} = \left(\frac{1}{4}\right)^{1/2} = \frac{1}{2} = 0.5 \qquad (4)$$

The second moment of x about the origin is 2.i.e.

$$\mu_2' = 2,$$

$$\mu_1' = \overline{X} = 1 \qquad (5)$$

$$\mu_2 = \sigma_x^2 = \mu_2' - (\mu_1')^2 = 1$$

$$\therefore \sigma_x = 1$$

$$\therefore \sigma_y = \frac{b_{yx} \times \sigma_x}{r} = \frac{1 \times 1}{1/2} = 2 \qquad (6)$$

Q. No. 6.a.Find Fourier integral representation of:

$$f(x) = x \quad 0 < x < a$$

$= 0 \quad x > a$

$f(-x) = f(x)$ *6*

Ans: Fourier integral:

$$f(x) = \frac{1}{\pi}\int_0^{\infty}\int_{-\infty}^{\infty} f(s)\cos\omega(s-x)\,d\omega\,ds \quad (1)$$

But f(x) is even function

$$\therefore f(x) = \frac{2}{\pi}\int_0^{\infty}\cos\omega x\left\{\int_0^{\infty} f(s)\cos\omega s\,ds\right\}d\omega \quad (2)$$

$$f(x) = \frac{2}{\pi}\int_0^{\infty}\cos\omega x\left\{\int_0^{a} s\cos\omega s\,ds\right\}d\omega \quad (3)$$

$$f(x) = \frac{2}{\pi}\int_0^{\infty}\cos\omega x\left[\frac{s\sin\omega s}{\omega}+\frac{\cos\omega s}{\omega^2}\right]_0^a d\omega \quad (4)$$

$$f(x) = \frac{2}{\pi}\int_0^{\infty}\cos\omega x\left[\frac{a\sin a\omega}{\omega}+\frac{\cos a\omega}{\omega^2}-\frac{1}{\omega^2}\right]d\omega$$

$$f(x) = \frac{2}{\pi}\int_0^{\infty}\cos\omega x\left[\frac{a\omega\sin a\omega}{\omega^2}+\frac{\cos a\omega}{\omega^2}-\frac{1}{\omega^2}\right]d\omega \quad (5)$$

$$f(x) = \frac{2}{\pi}\int_0^{\infty}\cos\omega x\left[\frac{a\omega\sin a\omega+\cos a\omega-1}{\omega^2}\right]d\omega \quad (6)$$

Q.No.6.b. In certain experiment to compare two type of pig-foods A and B are the following results of increasing weights were obtained.

Pig Number	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*
Increase in wt x kg by A	*49*	*53*	*51*	*52*	*47*	*50*	*52*	*53*
Increase in wt x kg by B	*52*	*55*	*52*	*53*	*50*	*54*	*54*	*53*

Assuming that the two sample of pigs are independent, can we conclude that food B is better than food A. 6

Ans:

	Food A			Food B	
x_1	$d_i = x_1 - 51$	d_i^2	x_2	$d_j = x_2 - 53$	d_j^2
49	-2	4	52	-1	1

53	2	4	55	2	4
51	0	0	52	-1	1
52	1	1	43	0	0
47	-4	16	50	-3	9
50	-1	1	54	1	1
52	1	1	54	1	1
53	2	4	53	0	0
Total	-1	31	Total	-1	17

(2)

$$\overline{x_1} = 51 + \frac{\sum d_i}{n} \qquad \overline{x_2} = 53 + \frac{\sum d_j}{n}$$ *(3)*

$$\overline{x_1} = 50.875 \qquad \overline{x_2} = 52.875$$

$$\sum\left(x_1 - \overline{x_1}\right)^2 = \sum d_i^2 - \frac{\left(\sum d_i\right)^2}{n} = 30.875$$

$$\sum\left(x_2 - \overline{x_2}\right)^2 = \sum d_j^2 - \frac{\left(\sum d_j\right)^2}{n} = 16.875$$

Null Hypothesis: $\mu_1 = \mu_2$

Alternative hypothesis: $\mu_1 < \mu_2$

Test Statistics: $s_p = \sqrt{\dfrac{\sum\left(x_i - \overline{x}_1\right)^2 + \sum\left(x_j - \overline{x}_2\right)^2}{n_1 + n_2 - 2}} = \sqrt{3.41}$ *(4)*

$$S.E. = s_p \times \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 0.92$$

$$t = \frac{\overline{x}_1 - \overline{x}_2}{S.E.} = -2.17$$ *(5)*

$$|t| = 2.17$$

LOS: $\alpha = 5\%$

Critical value: At $\nu = 8+8-2 = 14 \to t_\alpha = 1.76$ *(6)*

Decision: The null hypothesis is rejected.

Q.No.6.c. A rectangular metal plate with insulated surfaces is of width 'a' and so long as compared to its width that it can be considered infinite in length without introducing an appreciable error. If the two long edges x = 0 and x = a and as well as the one short edge are kept at 0^0 C and temp of the other edge y = 0 is given by u = k x for 0< x < a/2

=k (a-x) for a/2< x < a

Find the temperature u(x, y) at any point (x, y)of the plate. *8*

Ans: In steady state 2 dim heat flow eq.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{------------(1)}$$

$$u(x, y) = (c_1 \cos mx + c_2 \sin mx)(c_3 e^{my} + c_4 e^{-my}) \quad \text{------------(2)}$$ *(1)*

(i) u →0when y →∞ for all x ,putting in (2)

$$0 = (c_1 \cos mx + c_2 \sin mx)(c_3 e^{my})$$

$$c_3 = 0$$

$$u = (c_1 \cos mx + c_2 \sin mx)(c_4 e^{-my}) \quad \text{-----------(3)}$$ *(2)*

(ii) when x = 0 u = 0

$$0 = (c_1) c_4 e^{-my}$$

$c_1 = 0$ putting in(3)

$$u = (c_2 \sin mx) c_4 e^{-my}$$

$$u = c_5 \sin mx e^{-my}, (c_2 c_4 = c_5) \quad \text{--------------(4)}$$ *(3)*

(iii)when x = a , u= 0 putting in (4)

$$0 = c_5 \sin ma e^{-my}$$

$$\therefore c_5 = 0 \text{ or } \sin ma = 0 \tag{4}$$

But $c_5 = 0$ gives trivial solution (From eq 4.)

$$\therefore \sin ma = 0$$

$$\therefore ma = n\pi \quad \text{where } n = 1, 2, 3, ...$$

$$\therefore m = \frac{n\pi}{a} \text{ putting in eq. (3)}$$

$$u = c_5 \sin\frac{n\pi x}{a} e^{-n\pi y/a}$$

$$u = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{a} e^{-n\pi y/a} \quad \text{-----------------(5)} \tag{5}$$

(iv)when y= 0 ,u = u(x,0)=kx for 0< x < a/2

=k(a-x) for a/2< x < a

$$u = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{a} \tag{6}$$

is Fourier Half Range sine series for function

u= kx for 0< x < a/2

= k(a-x) for a/2< x < a

$$b_n = \frac{2}{a}\int_0^a f(x)\sin\left(\frac{n\pi x}{a}\right)dx$$

$$b_n = \frac{2}{a}\int_0^{a/2} kx\sin\left(\frac{n\pi x}{l}\right)dx + \int_{a/2}^{a} k(a-x)\sin\left(\frac{n\pi x}{l}\right)dx \tag{7}$$

$$b_n = \frac{4ka}{\pi^2}\left[\frac{1}{n}\cdot\sin\frac{n\pi}{2}\right]$$

$$u = \sum_{n=1}^{\infty}\frac{4ka}{\pi^2}\left[\frac{1}{n}\cdot\sin\frac{n\pi}{2}\right]\sin\frac{n\pi x}{a} \tag{8}$$

Q.No. 7.a. *Show that the set of functions* $\frac{\sin x}{\sqrt{\pi}}, \frac{\sin 2x}{\sqrt{\pi}}, \frac{\sin 3x}{\sqrt{\pi}}, ...$ *form a normal set in the interval [-π,π]* 6

Ans: $f_n(x) = \frac{\sin nx}{\sqrt{\pi}}, n = 1, 2, 3, ---$ *(1)*

$$\int_{-\pi}^{\pi} f_m(x) f_n(x) dx = \int_{-\pi}^{\pi} \frac{\sin mx}{\sqrt{\pi}} \frac{\sin nx}{\sqrt{\pi}} dx \quad (2)$$

$$\int_{-\pi}^{\pi} f_m(x) f_n(x) dx = \frac{-1}{2\pi} \int_{-\pi}^{\pi} [\cos(m+n)x - \cos(m-n)x] dx \quad (3)$$

$$\int_{-\pi}^{\pi} f_m(x) f_n(x) dx = \frac{-1}{2\pi} \left[\frac{\sin(m+n)x}{m+n} - \frac{\sin(m-n)x}{m-n} \right]_{-\pi}^{\pi}$$

Case(i): $m \neq n$

$$\int_{-\pi}^{\pi} f_m(x) f_n(x) dx = 0 \quad (4)$$

Case(ii): $m = n$

$$\int_{-\pi}^{\pi} [f_n(x)]^2 dx = \int_{-\pi}^{\pi} \left[\frac{\sin nx}{\sqrt{\pi}} \right]^2 dx = \frac{1}{\pi} \int_{-\pi}^{\pi} \sin^2 nx dx$$

$$\int_{-\pi}^{\pi} [f_n(x)]^2 dx = \frac{1}{\pi} \int_{-\pi}^{\pi} \left[\frac{1}{2} - \frac{\cos 2x}{2} \right] dx \quad (5)$$

$$\int_{-\pi}^{\pi} [f_n(x)]^2 dx = 1 \quad (6)$$

Q.No.7.b. *A machine produced 20 defective articles in batch of 400.After overhauling it produced 10 defectives in a batch of 300.Has the machine improved?* 7

Ans: Null Hypothesis: $p_1 = p_2$

Alternative hypothesis: $p_2 < p_1$ *(1)*

Test Statistics:

P_1=proportion of defective articles before overhauling $= \frac{X_1}{n_1} = \frac{20}{400} = 0.05$

P_2=proportion of defective articles before overhauling $= \frac{X_2}{n_2} = \frac{10}{300} = 0.033$ *(3)*

$p_1 - p_2 = 0.017$

$p = \frac{X_1 + X_2}{n_1 + n_2} = 0.043$ *(4)*

$q = 1 - p = 0.957$

$S.E. = \sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = 0.015$ *(5)*

$Z = \frac{p_1 - p_2}{S.E.} = 1.13$ *(6)*

LOS: $\alpha = 5\%$

Critical value: At $\alpha = 5\% \rightarrow Z_\alpha = 1.96$

$$|z| = 1.13 < Z_\alpha \quad (7)$$

Decision: The null hypothesis is accepted.

Q.No.7.c.A survey of 320 families with 5 children revealed the distribution of boys as given belo.Is the result consistent with the hypothesis that male and female births are equally probable? 7

Ans: Data is not given in problem (wrong question)

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