APPLIED MATHEMATICS-III

Q. 1) a) *If* 05

$\int_0^\infty e^{-2t}.\sin(t+\alpha).\cos(t+\alpha)dt = \frac{3}{8}$, *then find α.*

Ans: L(sin(t+α).cos(t-α))=$L\left(\frac{\sin 2t+\sin 2\alpha}{2}\right)=\frac{1}{2}\left[\frac{2}{s^2+4}+\sin 2\alpha.\frac{1}{s}\right]=\frac{3}{8}$ 01

$\left[\frac{2}{s^2+4}+\sin 2\alpha.\frac{1}{s}\right]=\frac{3}{4}$ 02

For s = 2, $\left[\frac{2}{4+4}+\sin 2\alpha.\frac{1}{2}\right]=\frac{3}{4}$

So, sin2α = 1 03

2α = π/2

α =π/4. 05

b) *Prove that every Hermitian matrix A can be express as B + iC, where B is real symmetric and C is real skew symmetric matrix.* 05

Ans: A = B + iC 01

$=\frac{1}{2}\{(A+\bar{A})\}+i\frac{1}{2i}\{(A+\bar{A})\}$

If Z & $\bar{Z}$ be complex and complex conjugates of each other then

$\frac{1}{2}(z+\bar{z})$ *and* $\frac{1}{2i}(z-\bar{z})$ *are real* 02

$B^T=\frac{1}{2}\{(A+\bar{A})\}^T=\frac{1}{2}\{(A^T+\bar{A}^T)\} = \frac{1}{2}\left\{\left(A^{\theta^T}+A^{\theta^T}\right)\right\}$ since A is Hermitian is given.

$=\frac{1}{2}\{(A+\bar{A})\} = B$

So, B is symmetric. 03

Similarly we can prove, $C^T=-C$ i.e. C is skew – symmetric. 05

c) *If f(z) is an analytic function, prove that* 05

$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]\log|f'(z)|=0$

Ans: 01

F(z) = u + iv

F'(z) = u_x + I V_x

$|f'(z)| = \sqrt{ux^2 + vx^2}$

$log|f'(z)| = log\sqrt{ux^2 + vx^2}$

$log|f'(z)| = \frac{1}{2} log|f'(z)|^2$

But, $|f'(z)|^2$=f'(z).f'($\bar{z}$)

$log|f'(z)| \ = \frac{1}{2} \log$ [f'(z).f'($\bar{z}$)] 02

$= \frac{1}{2} [logf(z) + \ logf(\bar{z})]$

Now,

$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) log|f'(z)| = 4\frac{\partial^2}{\partial z \partial \bar{z}} log|f'(z)|$

03

$-\ 4\frac{\partial^2}{\partial z \partial \bar{z}} [\frac{1}{2} log|f'(z)| + \frac{1}{2} log|f'(\bar{z})|$

$= 2\frac{\partial^2}{\partial z \partial \bar{z}} [log|f'(z)| + 2\ \frac{\partial^2}{\partial z \partial \bar{z}} log|f'(\bar{z})|$ 05

$=2\frac{\partial}{\partial z}\left(\frac{\partial}{\partial \bar{z}}\log(f'(z))\right) + 2\frac{\partial}{\partial z}\left(\frac{\partial}{\partial \bar{z}}\log(f'\bar{z})\right)$

$=2\frac{\partial}{\partial z}(0) + \ 2\frac{\partial}{\partial z}(0) = 0$

d) *Evaluate* $\int_c \left(z^2 + z\right) dz$ *along the line joining (1,-1), (2,3)* 05

Ans:

$\frac{y+1}{-1-1} = \frac{x-3}{3-2}$ 01

$\frac{y+1}{-2} - \frac{x-3}{1}$

Y + 1 = - 2 (x - 3)

Y + 1 = - 2x + 6

Y + 2x = 5

Y = 5 – 2x

Dy = -2 dx 02

Dz= dx + idy = dx = 2idx = (1-2i) dx

I = $\int_1^2 (x+iy)^2 + (x+iy)(dx+idy)$ 04

$= \int_1^2 (x^2 - y^2 + 2ixy) + (x+iy)(dx+idy)$

Sub. Y = 5 – 2x we get,

05

I = -75/6 + 32i/3

Q. 2) a) *Find the image of the triangle whose vertices are i, 1+i, 1-i under the transformation w = 3z +4 -2i.* 06

Ans:

U + iv=3(x+iy) +4 -2i

02

=3x + i3y + 4 – 2i

(3x+4) +i(3y-2)

Here u = 3x + 4 and v = 3y -2.

06

Solving we get vertices of triangle are:

(4,1) (7,1) and (7,-5)

b) *Find the inverse Laplace transform of* $\cot^{-1}\left(\frac{2}{s^2}\right)$. 07

Ans: $\cot^{-1} s = \frac{\pi}{2} - \tan^{-1} s$ 01

So,

$\cot^{-1}\frac{2}{s^2} = \frac{\pi}{2} - \tan^{-1}\frac{2}{s^2}$

Let

$\varphi(s) = \frac{\pi}{2} - \tan^{-1}\frac{2}{s^2}$

$\varphi'(s) = \frac{1}{1+\frac{4}{s^4}}\left(\frac{-4}{s^5}\right) = \frac{-4s}{s^4+4}$

So, $\frac{1}{t}L^{-1}(\varphi'(s)) = \frac{-4}{t}L^{-1}\left(\frac{s}{s^4+4}\right)$ 02

$= \frac{-4}{t}L^{-1}\left[\frac{s}{(s^2+2)^2-2s^2}\right]$

$= \frac{-4}{t}\frac{1}{4}L^{-1}\left[\frac{1}{s^2-2s+2} - \frac{1}{s^2+2s+2}\right]$

03

$= \frac{-1}{t}L^{-1}\left[\frac{1}{(s-1)^2+1} - \frac{1}{(s+1)^2+1}\right]$

$= \frac{-1}{t}e^{t}L^{-1}\left(\frac{1}{s^2+1}\right) - e^{-t}L^{-1}\left(\frac{1}{s^2+1}\right)$

05

$= -1/t\ (e^{t}\sin t - e^{-t}\sin t)$

07

$=(-2\sin t\ \sinh t)/t.$

c) *Find non singular matrices P and Q such that PAQ is in normal form. Also find rank of A , and A^{-1}.* 07

Where

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}.$$

Ans:

$A = I_3\ A\ I_3$ 01

$$\begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

R13

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

R2 – 3R1, R3 – 2 R1 02

$$\begin{bmatrix}1 & 2 & 3\\0 & -5 & -7\\0 & 3 & 3\end{bmatrix} = \begin{bmatrix}0 & 0 & 1\\0 & 1 & -3\\1 & 0 & 2\end{bmatrix} A \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

C2 – 2 C1, C3 – 3 C1

04

$$\begin{bmatrix}1 & 0 & 0\\0 & 5 & 7\\0 & -3 & -3\end{bmatrix} = \begin{bmatrix}0 & 0 & 1\\0 & 1 & 3\\1 & 0 & -2\end{bmatrix} A \begin{bmatrix}1 & -2 & -3\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

05

SO, $\zeta(A) = 2$

$$A^{-1} = \frac{1}{6}\begin{bmatrix}-1 & 3 & -1\\-7 & 3 & 5\\5 & -3 & -1\end{bmatrix}$$ 07

Q.3) a) *Show that the vectors x1, x2. X3 are linearly independent and the vector x4 depends upon them, where, x1 = (1,2,4), x2=(2,-1,3), x3=(0,1,2), x4 = (-3, 7, 2).* 06

Ans:

Ax1 + b x2 + c x3 = 0

A (1,2,4) + b(2,-1,3)+c(0,1,2)=0 01

A + 2b + 0c =0

2a –b + c =0

4a + 3b + 2c =0

AX =0 02

$$\begin{bmatrix}1 & 2 & 0\\2 & -1 & 1\\4 & 3 & 2\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

R2 – 2R1, R3 – 4 R1

04

$$\begin{bmatrix}1 & 2 & 0\\0 & -5 & 1\\0 & -5 & 2\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix} - \begin{bmatrix}0\\0\\0\end{bmatrix}$$ 05

R3-R2

$$\begin{bmatrix}1 & 2 & 0\\0 & -5 & 1\\0 & 0 & 2\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

SO, X1, X2, X3 are linearly independent. 06

But for X4 = (-3,7,2) given system becomes dependent.

So, X4 depend on them.

b) *Find the inverse Laplace transform by using convolution theorem of the function* $\frac{1}{s\sqrt{s+4}}$. 07

Ans: $L^{-1}\left(\frac{1}{(s+4)^{1/2}}\right) = e^{-4u}\frac{u^{-1/2}}{\sqrt{1/2}}$ 01

And $L^{-1}\left(\frac{1}{s}\right) = 1$

02

$L^{-1(\varphi 1.\varphi 2)} = L^{-1}\left(\frac{1}{s}.\frac{1}{(s+4)^{1/2}}\right)$

$$= \int_0^t \frac{e^{-4u}u^{-\frac{1}{2}}}{\sqrt{\pi}}(1)\,du$$

04

Put $4u = x^2$

$du = \frac{x\,dx}{2}, \sqrt{u} = x/2$ 05

$L^{-1}\left(\frac{1}{s}.\frac{1}{(s+4)^{1/2}}\right) = \int_0^{2\sqrt{t}} \frac{e^{-x^2}}{\sqrt{\pi}}\,dx$

07

$= 1/2\mathrm{erf}\,(2\sqrt{t})$

c) *State Residue theorem. Hence evaluate* 07

$$\int_0^{2\pi} \frac{d\theta}{3 - 2\cos\theta + \sin\theta}.$$

Ans: Statement: If f(Z) is an analytic function everywhere except some finite 01 number of points then

$$\int f(z)dz = 2\pi i(sum\ of\ the\ residues)$$

03

Let I $=\int_c \frac{dz}{iz}\frac{1}{3-2\frac{z^2+1}{2z}+\frac{z^2-1}{2iz}}$

$$= \int_c \frac{dz}{iz}\frac{2iz}{6iz-2i(z^2+1)+(z^2-1)}$$

05

$$= \int_c \frac{dz}{iz}\frac{2iz}{6iz-2iz^2-2i+(z^2-1)}$$

$$= \int_c \frac{1}{(1-2i)z^2+6iz-(1+2i)}dz$$

07

So, I = π(2+i)

Q.4) a) *Find the analytic function f(z) = u + iv such that u+v =* $\frac{sin2x}{cosh2y\quad cos2x}$ 06

Ans:

F(z) = u + iv

02

if(z) = ui – v

(1+i)f(z)= (u-v) +i(u+v)

= U + iV

V = u+v $= \frac{sin2x}{cosh2y-cos2x}$

Vx $=\frac{2cosh2ycos2x-2}{(cosh2y-cos2x)^2}$

04

Vy $=\frac{-2sinhysin2x}{(cosh2y-cos2x)^2}$

By Milne Thompson's method,

(1+i) f'(z) = Vy +iVx

= □1(z,0) +i□2(z,0)

(1+i) f(z) = $\int 0 + i\frac{2(cos2z\quad 1)}{(1-cos2z)^2}dz$

$= -2i \int \frac{dz}{1 \quad cos2z}$

05

$= -i \int cosec^2 z\, dz$

= I cotz + c'

So, f(z) = $\frac{1}{1+i} cotz + c.$ 06

b) *Express the following function as Heaviside's unit step function and find it's Laplace transform* 07

$f(t) = \cos t, 0 < t < \pi$
$= \cos 2t, \pi < t < 2\pi$
$= \cos 3t, t > 2\pi$

Ans: F(t)= cost(H(t) – H(t-π)) + cos2t[H(t-π)-H(t-2π)]+cos3t[H(t-2π] 01

= cost H(t) + (cos2t – cost) H(t-π) + (cos3t – cos2t) H(t-2π) 03

So, Lf(t) = L(cost) + $e^{-\pi s}$ L (cos2(t+π)-cos(t+π)) + $e^{-2\pi s}$L[cos3(t+2π)-cos2(t+2π)] 04

06

=L(cost) + $e^{-\pi s}$(cos2t + cost) + $e^{-2\pi s}$ (cos3t – cos2t)

$= = \frac{s}{s^2+1} + e^{-\pi s}[\frac{s}{s^2+4} + \frac{s}{s^2+1}] + e^{-2\pi s}\left[\frac{s}{s^2+9} - \frac{s}{s^2+4}\right]$

07

c) *Compute $A^9 – 6 A^8 + 10A^7 – 3A^6 + A + I$* 07

Where

$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 3 & 1 \\ - & 0 & 2 \end{bmatrix}$

Ans: $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 3 & 1 \\ - & 0 & 2 \end{bmatrix}$ 02

Characteristic equation is:

[A-λI][X]=[0]

$\lambda^3 - 6\lambda^2 + 10\lambda - 3 = 0$

By Cayley Hamilton theorem, 04

$A^3 - 6A^2 + 10A - 3 = 0$

SO, $A^9 - 6A^8 + 10A^7 - 3A^6 + A + I$

$= A^6(A^3 - 6A^2 + 10A - 3) + A + I$

06

$= A^6(0) + A + I$

$= A + I$

$= \begin{bmatrix} 2 & 2 & 3 \\ -1 & 4 & 1 \\ 1 & 0 & 3 \end{bmatrix}$ 07

Q. 5) a) *Evaluate the following integral by using Laplace transform* 06

$$I = \int_0^\infty e^{-t}\left[t\int_0^t e^{-4u}.\cos u du\right]dt$$

Ans: $L(\cos t) = \frac{s}{s^2+1}$ 02

$L(e^{-4t}\cos t) = \frac{s+4}{(s+4)^2+1}$

$L(e^{-4t}\cos t) = \frac{s+4}{(s+4)^2+1}$

$L\left(\int_0^t e^{-4u}\cos u du = \frac{1}{s}\left[\frac{s+4}{s^2+8s+17}\right]\right)$

04

$L\left(t\int_0^t e^{-4u}\cos u du = -\frac{d}{ds}\frac{1}{s}\left[\frac{s+4}{s^2+8s+17}\right]\right)$

$= -\left[\frac{(s^3+8s^2+17s)(1)-(s+4)(3s^2+16s+17)}{(s^3+8s^2+17s)^2}\right]$

Sub. S =1,

06

$= -\left(\frac{26-180}{(26)^2}\right) = 0.22$

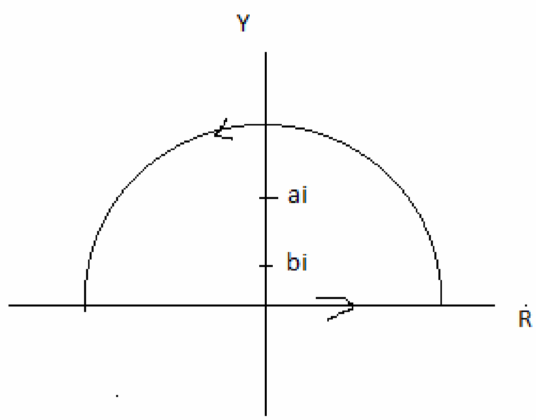
b) *Evaluate:* 07

$$\int_{-\infty}^{\infty} \frac{\cos x}{(x^2+a^2)(x^2+b^2)}dx, a>0, b>0.$$

Ans: $I = \int_{-\infty}^{\infty} \frac{\cos z}{(z^2+a^2)(z^2+b^2)}dz$ 02

Here poles of the integral are:

Z = ai, -ai, bi, - bi



The poles ai and bi lies inside the close curve C

Residue of f(z) at z = ai = $\lim_{z\to ai}(z-ai)\frac{\cos z}{(z-ai)(z+ai)(z^2+a^2)}$

$$= \frac{\cos ai}{(2ai)(-a^2+b^2)}$$

$$= \frac{\cosh a}{(2ai)(-a^2+b^2)}$$

05

Residue of f(z) at z = bi = $\lim_{z\to bi}(z-bi)\frac{\cos z}{(z-bi)(z+bi)(z^2+b^2)}$

$$= \frac{\cos bi}{(2bi)(-b^2+a^2)}$$

06

$$= \frac{\cosh b}{(2bi)(-b^2+a^2)}$$

By Residue theorem,

I = 2πi (sum of the residues)

$$= 2\pi i\left[\frac{\cosh a}{(2ai)(-a^2+b^2)} + \frac{\cosh b}{(2bi)(-b^2+a^2)}\right]$$ 07

$$= \frac{\pi}{a^2-b^2}\left[\frac{e^{-b}}{b} - \frac{e^{-a}}{a}\right]$$

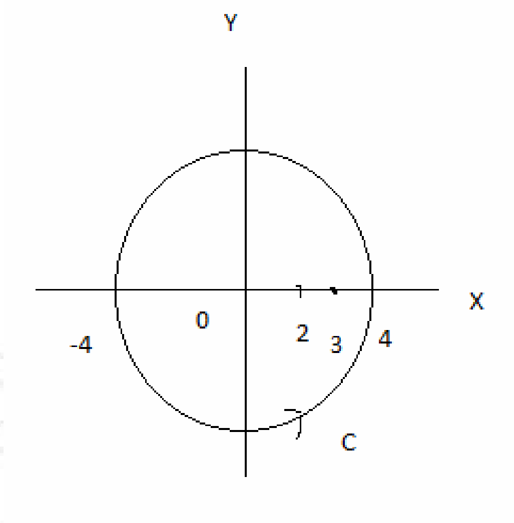
c) *State Cauchy's Integral formula and hence evaluate* 07

$$\int_{C:|z|=4} \frac{\sin \pi z^2 + \cos \pi z^2}{(z-2)(z-3)}\,dz$$

Ans: Statement: If f(z) is analytic everywhere except at z = a, then 02

$$\int \frac{f(z)}{z-a}\,dz = 2\pi i\, f(a)$$

Here z = 2 and z=3 are the poles which ling inside C



$$\frac{1}{(z-2)(z-3)} = \frac{A}{z-2} + \frac{B}{z-3}$$ 04

1 = A(z -3) + B(z – 2)

Solving we get, A = -1 and B = 1

$$\frac{1}{(z-2)(z-3)} = \frac{-1}{z-2} + \frac{1}{z-3}$$

$$\int \frac{\sin\pi z^2 + \cos\pi z^2}{(z-2)(z-3)} dz = \int \frac{\sin\pi z^2 + \cos\pi z^2}{(z-3)} dz - \int \frac{\sin\pi z^2 + \cos\pi z^2}{(z-2)} dz$$ 06

$$= 2\pi i \, f(3) - 2\pi i \, f(2)$$

07

$$= 2\pi i \, (-1) - 2\pi i \, (1) = -4\pi i$$

Q. 6) a) *Find all possible Laurent's expansions of the function* 06

$$f(z) = \frac{7z-2}{z(z-2)(z+1)} about z = -1.$$

Ans: $\frac{7z-2}{z(z-2)(z+1)} = \frac{A}{z} + \frac{B}{z-2} + \frac{c}{Z+1}$ 01

$= \frac{1}{z} + \frac{2}{z-2} - \frac{3}{Z+1}$

$= \frac{1}{(z+1)-1} + \frac{2}{(z+1)-3} - \frac{3}{z+1}$ 02

(i) When |z+1|<1 we write,

$F(z) = \frac{-3}{z+1} - \frac{1}{1-(z+1)} - \frac{2}{3-(z+1)}$

When |z+1|<1 then obviously |z+1|<3

$F(z) = \frac{-3}{z+1} - \frac{1}{1-(z+1)} - \frac{2}{\frac{2}{3}(1-\frac{z+1}{3})}$

$= \frac{-3}{z+1} - [1+(z+1)+(z+1)^2+\cdots\ldots] - \frac{2}{3}\left[1+\frac{z+1}{3}+\frac{z+1^2}{9}+\cdots\ldots\right]$

04

(ii) 1 < |z+1|< 3

$F(z) = \frac{-3}{1+z} + \frac{1}{(z+1)\left(1-\frac{1}{z+1}\right)} - \frac{2}{3\left(1-\frac{z+1}{3}\right)}$

$= \frac{-3}{1+z} + \frac{1}{(z+1)}\left[1+\frac{1}{z+1}+\frac{1}{z+1^2}+\cdots\ldots\right] - \frac{2}{3}\left[1+\frac{z+1}{3}+\frac{z+1^2}{9}+\cdots\ldots\right]$

(iii)|z+1|> 3

$F(z) = \frac{-3}{1+z} + \frac{1}{(z+1)\left(1-\frac{1}{z+1}\right)} - \frac{2}{(z+1)\left(1-\frac{3}{z+1}\right)}$

$= \frac{-3}{1+z} + \frac{1}{(z+1)}\left[1+\frac{1}{z+1}+\frac{1}{z+1^2}+\cdots\ldots\right] - \frac{2}{z+1}\left[1+\frac{3}{z+1}+\frac{9}{z+1^2}+\cdots\ldots\right]$ 06

b) *Solve the following equation by using Laplace transform* 07

$\frac{dy}{dt} + 2y + \int_0^t y\,dt = \sin t$ *given y(0)=1.* 02

Ans: Taking laplace transform on both sides,

L(y') + 2 L(y) + $L\left(\int_0^t y dt\right) = L(sint)$

s$\bar{y} - 1 + 2\bar{y} + \frac{1}{s}\bar{y} = \frac{1}{s^2+1}$ 04

$\left(s + 2 + \frac{1}{s}\right)\bar{y} = \frac{1}{s^2+1} + 1 = \frac{s^2+1+1}{s^2+1}$

$\bar{y} = \frac{s(s^2+2)}{(s+1)^2(s^2+1)}$ 05

So, y = $L^{-1}\left(\frac{s(s^2+2)}{(s+1)^2(s^2+1)}\right)$

$= L^{-1}\left[\frac{1}{s+1} - \frac{3}{2}\frac{1}{(s+1)^2} + \frac{1}{2}\frac{1}{(s^2+1)}\right]$

07

Y = $e^{-t} - \frac{3}{2}te^{-t} + \frac{1}{2}sint$

c) *Find the bilinear transformation which maps z=0, I, -2i onto w = -4i, ∞,0 respectively. Also obtain fixed points of the transformation.* 07

Ans: $w = \frac{ai+b}{ci+d}$ 01

Z = 0, I, -2i and w = -4i, ∞, 0

$-4i = \frac{b}{d}, \frac{1}{0} = \frac{ai+b}{ci+d}, \quad 0 = \frac{-2ai+b}{-2ci+d}$

Solving,

B = -4di, d = - ci

B = -4c, a = 2ci 03

$w = \frac{ai + b}{ci + d}$

$= \frac{2ciz - 4c}{cz - ci} = \frac{2iz - 4}{z - i} = -\left(\frac{2z+4i}{iz+1}\right)$ 05

Sub. w = z

$Z = -\left(\frac{2z+4i}{iz+1}\right)$

So, z = -4/I, -i are fixed points. 07

Q. 7) a) *Show that $u = e^{-2xy}\sin(x^2 - y^2)$ is harmonic. Find the corresponding analytic function f(z) and also find the harmonic conjugate function v.* 06

Ans: Let $u = e^{-2xy}\sin(x^2 - y^2)$ 01

So, uxx + uyy = 0

U is Harmonic.

F'(z) = ux + ivx 02

= ux –iuy

F(z) = $\int ux - iuy$

By milne Thomson's method, 04

F(z) = $-ie^{-iz^2} + c$

So, v = $-e^{-2xy}\cos(x^2 - y^2)$ 06

b) *Find* $L(erf\sqrt{t})$ *and hence find* $L(t.erf\,2\sqrt{t})$ 07

Ans:

$$erf\,(t) = \frac{2}{\sqrt{\pi}} \int_0^t e^{-x^2}\,dx$$

$$so, erf\,(\sqrt{t}) = \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{t}} e^{-x^2}\,dx$$

$$= \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{t}} \left(1 - x^2 + \frac{x^4}{2!} - \ldots\ldots\right) dx$$ 02

$$= \frac{2}{\sqrt{\pi}} \left[t^{1/2} - \frac{t^{3/2}}{3} + \frac{t^{5/2}}{5.2!} - \ldots\ldots \right]$$ 03

$so,$

$$L\left(erf\,(\sqrt{t})\right) = \frac{1}{s^{3/2}} \left[1 - \frac{1}{2}\frac{1}{s} + \frac{1}{2}\frac{3}{4}\frac{1}{s^2} - \ldots\ldots \right]$$

$$= \frac{1}{s^{3/2}} (1 + 1/s)^{-1/2}$$ 05

$$= \frac{1}{s\sqrt{s+1}}$$

$$L(erf\,(2\sqrt{t})) = L(erf\,\sqrt{4t})$$

$$= \frac{1}{4} \frac{1}{s/4\sqrt{s/4+1}}$$ 06

$$= \frac{2}{s\sqrt{s+4}}$$

$$L(terf\,2\sqrt{t}) = -\frac{d}{ds}\left(2\left(s^3 - 4s^2\right)^{-1/2}\right)$$

$$= -2(-\frac{1}{2})\left(s^3 - 4s^2\right)^{-3/2}\left(3s^2 + 8s\right)$$

$$= \frac{3s+8}{s^2(s+4)^{3/2}}$$ 07

c) $If N = \begin{bmatrix} 0 & 1+2i \\ -1+2i & 0 \end{bmatrix}$, *then show that (I-N)(I+N)$^{-1}$ is a unitary matrix.* 07

Ans: I-N = $\begin{bmatrix} 1 & -(1+2i) \\ 1-2i & 1 \end{bmatrix}$ 02

03

$$I + N = \begin{bmatrix} 1 & (1+2i) \\ -1+2i & 1 \end{bmatrix}$$

04

$$(I+N)^{-1} = \frac{1}{6}\begin{bmatrix} 1 & -(1+2i) \\ 1-2i & 1 \end{bmatrix}$$

$$A = (I-N)(I+N)^{-1} = \frac{1}{6}\begin{bmatrix} -4 & -2(1+2i) \\ 2(1-2i) & -4 \end{bmatrix}$$

06

$$\bar{A} = \frac{1}{6}\begin{bmatrix} -4 & -2(1-2i) \\ 2(1+2i) & 4 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} -2 & -(1+2i) \\ (1-2i) & -2 \end{bmatrix}$$

$$A^{\theta} - \frac{1}{3}\begin{bmatrix} -2 & (1+2i) \\ -(1-2i) & -2 \end{bmatrix}$$ 07

So we get, $AA^{\theta} = I$